U 63074 (Pages : 2) Name...............................

Reg. No............................

SECOND SEMESTER M. S c. DEGREE EXAMINATION, JUNE 2019

(CUCSS)

Mathematics

MT 2C 07—ALGEBRA—II

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Verify whether Z/4Z is an integral domain.
2. Find *irr (NI 2 + .; Q).*
3. Show that Q(-4, -M is a simple extension.
4. Find the degree [(0,(0): Q] where co is a non-real cube root of unity.
5. Let a = **1 +** 2i be a complex number. Show that R(a) = C.
6. Find all automorphisms of Q(,r).
7. Is f constructible. Justify your answer.
8. Verify whether F = {0,1,a, 1 + a} is a subfield of Zi where Z2 is an algebraic closure of Z2 containing a zero a of $x^2 + x + 1$.
9. Let a be an automorphism of Q('I,`/3) such that a(V2) = and a (,/a) = —Id. Find the fixed field of a.
10. Find the index {Q(f) • Q} •
11. List the elements of the Galois group G(QR/Q).
12. Describe an extension K of Q such that G(K/Q) is of order 3.
13. Verify whether $x^3 — 1$ is a cyclotomic polynomial.
14. Show that $x^3 — 1$ is solvable by radicals over Q.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any seven *questions.*
Each question carries 2 *weightage.*

15. Show that Z_{io}/I is a field where I is the ideal generated by 5 in Zicr

16. Verify whether the field C of complex numbers is an algebraic extension of the field CP of rational numbers.

17. Let a be an algebraic element over a field F and p c F(a). Show that 3 is algebraic over F.

18. Let a be algebraic over a field E. Verify whether a is algebraic over every subfield F of E.

19. Show that if F is a finite field of characteristic p then F is an extension of Z.

20. Let E be an extension of a field F and *a* E E be algebraic over F. Let n be an automorphism of E leaving F fixed and u(a) = 13. Show that *irr(a ;* F) = *irr((3;* F).

21. Show that Q(a, is a splitting field over Q.

22. Find all automorphisms of the field Q(i) where *i* is square root of — 1.

23. Let K be the splitting field of $(x^2 - 2)(x^2 + 2)$ over Q. Describe the Galois group G(K/Q).

24. Show that a regular 7-gon is not constructible by straight edge and compass.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries 4 *weightage.*

25. (a) Show that if F is a field then the polynomial ring F[x] is a principal ideal ring.

    (b) Show that an ideal *(p(x))* in F[x] is maximal if and only if *p(x)* is irreducible.

26. Let E be an extension of a field F and a E E be algebraic over F. Show that

    (a) There exists an irreducible polynomial *p(x)E* F[x] such that p(a) = 0.

    (b) If *q(x)* is also an irreducible polynomial such that q(a) = 0 then *p(x)* and *q(x)* have same degree.

27. (a) Let F be a finite field of *q* elements. Show that *q = p"* for some natural number n where *p* is the characteristic of F.

    (b) Show that if F and F' are finite fields having the same number of elements then F and F' are isomorphic.

28. (a) Define the nth cyclotomic polynomial (I) (x) over a field F.

    (b) Show that the Galois group of the 5th cyclotomic polynomial over Q is a cyclic group of order 4.

(2 x 4 = 8 weightage)