C 63075 (Pages 3) Name....................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS)

Mathematics

MT 2C 08—REAL ANALYSIS—II

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer all *questions.*
Each question carries 1 weightage.

1. Let A c R and let E > 0. Prove that there is an open set 0 containing A such that m* (0) m*(A) + E.
2. Let .M be a a-algebra and let A, B EM. Prove that AnB E
3. Prove that continuous functions are Lebesgue measurable.
4. Let *f* be a measurable function and let *f* = *g* a.e. Prove that *g* is measurable.
5. Let *f* be a non-negative measurable function. If A and B are measurable sets with A D B, then prove that $\underset{A}{.1} f\,dx > {}^{\mathrm{f}} f\,dx$.
6. If *f* is integrable, then prove that *f* is finite valued a.e.
7. If *f'* exists and bounded on *[a, b]*, then prove that *f* is of bounded variation on *[a, b]* .
8. Let *f* be an integrable function on *(a, b)*. Prove that the Lebesgue set of *f* contains all points in *(a, b)* at which *f* is continuous.
9. Let A, B be subsets of a set C, let A, B, C be elements of a ring R and let p. be a measure on R. If μ(A) = μ(C) < co, then prove that p (A n B) = μ (B).
10. Define complete measure and give an example of it.

li. Let A be a positive set with respect to a signed measure u on a measurable space [IX, *St* Prove that every measurable subset of A is a positive set.

12. Prove that the total variation of a signed measure on a measurable space IX, SJ1 is a measure on
13. Give an example to show that a Hahn decompsition is not unique.

Turn over

14. Let u be a measure on measure space 1[X, 8, µT. Give sufficient conditions on the measure µ and v so that there exists a non-negative measurable function *f* on X such that u(E) = j_E *f elpt* for each EE *S*.

(14 X 1 = 14 weightage)

Part B

Answer any seven *questions.*
Each question carries 2 *weightage.*

15. Let E_1 and E_2 be Lebesgue measurable sets. If E_i nE_2 =4), then prove thµt m(E_i uE_2)= m(E_1)+m(E_2).

16. Let {Ei} be a sequence of measurable sets. If E_1 p E_2 - - - and m(E_i) < co for each *i*, then prove that m(limE_1) = limm(E_i).

17. Let If, } be a sequence of measurable functions defined on the same measurable set. Prove that lim sup f_n is measurable.

18. For *x* E [0, 11, define

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational.} \end{cases}$$

Evaluate $\int_0^1 f\, dx$.

19. Let *f* be a finite valued monotone increasing function on [a, *b]*. Prove that *f is* continuous except on a set of points which is at most countable.

20. Let {a_n } be a sequence of non-negative numbers and for A c N, let µ(A) = $\sum_{nEA} a_n$. Show that µ is a complete measure on the measurable space [[N, P(11)J1.

21. Prove that Jordan decomposition of a signed measure is unique.

22. Prove that countable union of sets positive with respect to a signed measure µ is a positive set.

23. If v_i, v_2 are a-finite measures on a measurable space [X, S]] and v_1 « v_2 « µ, then prove that

$$\frac{d(v_i + v_2)}{d\mu} = dv_i \, \underline{dv_2}$$

24. Let *f* be an integrable function on (a, *b).* Prove that the function F (x) = $\int_a$ *N) dt* is absolutely continuous.

(7 X 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries **4** *weightage.*

25. Prove that the set of all Lebesgue measurable subsets of R is a a-algebra.

26. Let *f* and *g* be non-negative measurable functions. Prove that

$$f\,dx + jg\,dx = f(f + g)dx.$$

27 Ifμ is a-finite measure on a ring TZ, then prove that it has a unique extension to the a-ring S(R), where *S(R)* is the a-ring generated by R.

28. Let [[X, S, p.]] be a a-finite measure space and u be a measure on *S*. Prove that $v = v_o + v_1$, where v_o and v_1 be measure on *S* such that v_o 1 μ and v_1 << μ. Also prove that the decomposition is unique.

(2 x 4 = 8 weightage)