C 63078 **(Pages : 2)** **Name.................................**

Reg. No.............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS)

Mathematics

MT 2C **11—OPERATIONS** RESEARCH

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
EacA question carries a weightage of 1.

1. Is the function $f(x) = x^3, x \in R$, a convex function. Justify your claim.
2. Define basic feasible solution in linear programming problems.
3. Define the dual of a linear programming problem. Give an example for the same.
4. Define artificial variables. Describe the uses of artificial variables in solving linear programming problems.
5. Why do we introduce new variables in Linear programming problems?
6. What is canonical form of equations ?
7. Define slack and surplus variables in a linear programming problem.
8. What are simplex multipliers ?
9. Describe the general form of an integer linear programming problem in two dimensional space.
10. What is meant by loops in a transportation array ?
11. What is Caterer problem in operations research ?
12. Describe the minimum path problem in network analysis.
13. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
14. State the fundamental theorem of rectangular games.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. Prove that a basic feasible solution of a linear programming problem is a vertex of the convex set of feasible solutions.
16. Solve graphically the linear programming problem :

    Minimize $z = x_1 + 3x_2$ subject to $x_1 + x_2 > 3, -x_1 + x_2 < 2, x_1 - 2x_2 < 2, x_1 > 0, x_2 > 0$.

Turn over

17. Prove that the transportation problem has a triangular basis.
18. Describe the terms : chain, path, cycle, circuit and component with reference to graphs.
19. Show that if $\{x_i\}$ and $\{y_i\}$ are two flows in a graph, then $\{ax_i + byd$, where a and b are real constants, is also a flow.
20. Define spanning tree of a graph. Describe an algorithm for finding the spanning tree of minimum length of a graph.
21. Illustrate branch and bound method through an example.
22. Describe the two-person, zero-sum game.
23. Explain the terms mixed strategy, pure strategy and optimal strategies with reference to any matrix game.
24. State and prove the mini max theorem in theory of games.

(7 x 2 = 14 weightage)

Part C

Answer any two *questions.*
Each question carries a weightage of 4.

25. Solve the following problem using simplex method :

    Maximize $f(X) = 5x_1 + 3x_2 + x_3$ subject to the constraints $2x_1 + x_2 + x_3 = 3$, $—x_1 + 2x_1 = 4$, x_1 ?_ 0, $x_2 > 0$, x_3 **O.**

26. Using cutting plane method, solve the problem :

    Maximize $x_1 + x_2$ subject to $2x_1 < 3$, $2x_1 + 2x_2 > 5$, $—2x_1 + 2x_2 < 1$; x_1, x_2 non-negative integers.

27. Solve the transportation problem for minimum cost starting with the degenerate solution $x_{12} = 30$, $x_{21} = 40$, $x_{32} = 20$, $x_43 = 60$.

	D_1	D2	D3	
O_1	4	5	-2	30
O_2	4	1		40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

28. Examine the following pay-off matrix for saddle points. In the case if saddle point exists, find the optimal strategy and value of the game.

$$\begin{pmatrix} 2 & -1 & 2 \\ 1 & 0 & 1 \\ -2 & -1 & 2 \end{pmatrix}$$

(2 x 4 = 8 weightage)