

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2019

(CUCBCSS-UG)

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Types)

Answer all twelve questions.

I. Define a sequence.

2. Fill in the blanks • $\frac{d}{dx} \cosh^3(3x) = \underline{\hspace{2cm}}$

3. For what values of real numbers x , does the series $\sum_{n=1}^{\infty} \sin^n x$ converge ?

4. Fill in the blanks : The polar equation of the circle with centre origin and radius a is $\underline{\hspace{2cm}}$

5. Find the n^{th} term of the sequence $2, -2, 2, 2, \dots$ $\underline{\hspace{2cm}}$

6. Fill in the blanks : If $f(x, y) = 1 - \sinh(1 - xy)$, then $f_x(1, 1) = \underline{\hspace{2cm}}$

7. Fill in the blanks: If f is continuous on $[a, b]$, then $\lim_{c \rightarrow b} \int_c^b f(t) dt = \underline{\hspace{2cm}}$

8. Write explicitly the ratio test for the convergence of the series $\sum_{n=0}^{\infty} a_n$ $\underline{\hspace{2cm}}$

9. State alternating series test of Leibniz.

10. Define $f(x, Y)$ using limit.

11. The power series $\sum_{n=0}^{\infty} a_n (x-a)^n$ always converges to a_0 when $x = \underline{\hspace{2cm}}$

12. What do you mean by linearization of a function in two variables at a point.

(12 x 1 = 12 marks)

Turn over

Part B (Short Answer Types)

Answer any nine questions.

13. Evaluate $\int_0^1 \sinh^2 x \, dx$.
14. Test the convergence of the integral $\int_0^{1/2} \frac{1}{1-2x} \, dx$.
15. State the non-decreasing sequence theorem.
16. Describe the level surface of the function $f(x, y, z) = x^2 + y^2 + z^2 = 1$.
17. Graph the sets of points whose polar co-ordinates satisfy the condition $0 < r < 2$.
18. Evaluate $\int_0^1 \frac{x^3 \, dx}{\sqrt{4 + 9x^2}}$.
19. Find $\tanh x$, if $\cosh x = \frac{17}{15}$, $x > 0$.
20. Show that $\nabla^2 f = 0$ if $f(x, y) = \log \sqrt{x^2 + y^2}$.
21. Find a cylindrical co-ordinate equation for the surface $x^2 + y^2 - z^2 = 9$.
22. Find $\frac{1}{ar}$. $z = x + 2y$, $x = r \cos \theta$ and $y = r \sin \theta$.
23. Find $\lim_{n \rightarrow \infty} \frac{1}{2n+1}$.
24. Write the Maclaurin series for $\sin x$.

(9 x 2 = 18 marks)

Part C (Short Essay Types)

Answer any six questions.

25. Find the length of the curve $y = \frac{2\sqrt{2}}{3} x^{3/2} - 1$ from $x = 0$ to $x = 1$.
26. Find the limit of the function $f(x, y) = \frac{x^2 - xy}{\sqrt{x}}$ as (x, y) tends to $(0, 0)$.

27. Replace the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$ by equivalent Cartesian equation and draw the graph in Cartesian form.
28. Find a power series for $\log(1+x)$ and find the radius of convergence of that series.
29. Show that $\tan^{-1} x = 2 \frac{(1+x)^{-1}}{1-x^2}$.
30. Find the volume of the solid of revolution when the region between the parabola $x = y^2 + 1$ and the line $x = 3$ is revolved about the line $x = 3$.
31. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n - 1}{4^n}$.
32. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n (2x-1)^n}{n!}$.
33. Evaluate $\int_0^{\pi/4} r \cos h^4 dx$.

(6 x 5 = 30 marks)

Part D (Essay Types)*Answer any two questions.*

34. Show that the function $f(x, y) = \frac{2}{x^2 + y^2}$ when $(x, y) \neq (0, 0)$ and 0, otherwise is continuous everywhere except at the origin.
35. (a) Find the linearization of the function $f(x, y) = x^2 - xy + y^2/2 + 3$ at $(3, 2)$.
(b) Find the area of the region enclosed by the cardioid $r = 2(1 + \cos \theta)$.
36. Find the area of the surface generated by revolving the curve $y = x^3/9, 0 < x \leq 2$ about the x-axis.

(2 x 10 = 20 marks)