0 62652 (Pages : 4) Name

Reg. No..............................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
MAY 2019

B.Sc. Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**
Each question carries 1 mark.

Name the following :

1. The second central moment of a random variable X.

2. E $[e^{itx}]$ of a random variable X.

3. Probability distribution of the random variable X denoting the number of failures before the first success in an experiment with only two possible results success and failure.

Fill up the blanks :

4. Pearson's co-efficient of correlation between two random variables satisfying the linear relation 2x + 3y — 5 = 0 perfectly is ________

5. Probability density function of X following U [2, 5] = ________

6. X and Y are two random variables with bivariate m.g.f. function, $M_{x,y}(t_1, t_2)$ is expressed in terms of expectation as ________

7. The range of variation of beta distribution of second kind is ________

Write True *or* False :

8. m.g.f. exists for all the random variables.

9. Normal distribution is symmetric about its median.

10. Central limit theorem discusses the convergence of sum of random variables to normal distribution.

(10 x 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in* **one sentence.**
Each question carries 2 *marks.*

11. Find the mean of a random variable X with first moment about 4 is given as 7.

12. *f (x, y)* is the joint p.d.f. of two continuous random variables X and Y. Write any *two* of its properties.

13. Write the mean and variance of a binomial distribution with parameters (15, 0.3) .

14. Define covariance of X and Y.

15. If $p(x, y) = \frac{(2x + y)}{10}$, where $x = 0.1$ and y = 1.2 is a joint p.m.f. of X and Y, write the p.m.f. of X.

16. State Bernoulli's law of large numbers.

17. Define lognormal distribution.

(*7 x 2* = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. First three raw moments of X are — 1 , 55 and — 62.5. Obtain co-efficient of skewness based on moments.

19. State and prove Cauchy-Schwartz inequality.

20. For a random variable X, P (X = 3) = 2P (X = 4) = P (X = 5). Find V (X).

21. Obtain the m.g.f. of X following Poisson distribution with parameter A,.

22. The repair time of a machine follows exponential distribution with an average of 2 hours. What is the probability that the repair time will be more than 2 hours ?

(3 x 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 marks.

23. $f(x,y)=e^{-x-y}$, $X > 0$, $y > 0$ be the joint p.d.f. of (X, Y). Find the bivariate m.g.f. of (X, Y) and hence show that X and Y are independent.

24. Show that E (XY) = E (X) E (Y) need not imply X and Y are independent.

25. For two random variables X and Y, the joint p.d.f.

$f(x, y) = 2$
= 0, otherwise . Find E (X) and E (Y).

26. If X - N Y N (12, a2), obtain the distribution of aX + by when X and Y are independent.

27. If X - N(12, 4) . Obtain (i) P (X 5 20) ; (ii) **P (0 S X 24) ;** and (iii) P (IX - 121 > 8).

28. State and prove Tchebychev's inequality.

(4 x 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries **10** *marks.*

29. Let X and Y are two random variables with joint p.d.f. $f(x, y) = x + y$; $0 < x < 1$; $0 < y < 1$. Compute the coefficient of correlation between X and Y.

30. Obtain the m.g.f. of X following binomial distribution with parameters n and p. Hence state and prove the additive property of binomial distribution. If X and Y are independent binomial random variables with parameters (6, 0.5) and (4, 0.5) respectively, calculate P (X + Y 3).

Turn over

31. If X - (kt, a), prove that (a) $\frac{X-\mu}{a}$ N (0,1) ; (b) The quartile deviation of X is 0.6745 a.

32. State and prove Weak law of large numbers. Examine whether WLLN hold good for the sequence of random variables {X, }, i = 1, 2,... where P (X, = ± J2i - 1) = 0.5.

(2 x 10 = 20 marks)