C. 62588   (Pages : 4)   Name

Reg. No..............................

SECOND SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION, MAY 2019

B.Sc.—Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours   Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 mark.

1. Find $\lim\limits_{x \to 4} \dfrac{6^{\frac{x}{5}}}{co\ x} - I - 2$ 0'

2. Find absolute extrema of $y = x^2$ **on (0,** 2).

3. Find dy if y = 6 cos x^5.

4. Find the interval in which $y = x^3$ is concave up.

5. Suppose $\int_2^3 f(x)\, dx$ —6. Find - $\int_3^2 f(x)\, dx.$

6. Express the limit $\lim\limits_{\|\Delta\| \to 0} \sum_{k-1} (c_k^2 - 3c_k)\Delta x_k$ as an integral, where P is the partition of [- 7, 5] .

7. Define average value of a function f on [a, *b].*

8. Find all possible functions with derivative $y' = x^2$.

9. Evaluate $\int_0^{7c/2} \sin^2 x\, dx.$

10. Write the sum $\sum\limits_{k=1}^{5} k(3k+5)$ without sigma notation.

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11. State Mean Value Theorem.

12. Evaluate $\int_1^1 x^{2/3} \, dx$.

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Find $\lim_{t \to \underline{\quad}} \dfrac{2 - t + \sin t}{t + \cos t}$.

14. Evaluate $\displaystyle\sum_{k=1}^{9} k3$

15. Find the average value of the function $f(t) = \sin t$ on $[0, 2\pi]$ 1-

16. Express the solution of the initial value problem $\dfrac{dy}{dx} = \tan x, y\,(1) = 5$ as an integral.

17. Find $\dfrac{dy}{cx} \displaystyle\int_0^{\pi r} \cos t\,dt$.

18. Find absolute extrema values of $g\,(t) = 8t - t^4$ on $[-2, 1]$.

19. Suppose that f is continuous and that $\displaystyle\int_0^5 f(z)\,dz = 3$ and $\displaystyle\int_0^6 f(z)\,dz = 7$. Find $\displaystyle\int_5^{} f(z)\,dz$.

20. The region between the curve $y = \sqrt{x}$, $0 < x < 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

21. Find the work done by the force $F\,(x) = \dfrac{1}{x^2}N$ along x-axis from $x = 1$ m to $x = 10$ m.

22. Find $\lim_x \sin 2x$

23. Evaluate $\int_{1}^{7/3} 2\sec^t x\,dx.$

24. Find the interval in which $f(x) = -x^2 - 3x + 3$ is increasing and decreasing.

(9 x 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 marks.

25. Find asymptotes of the graph of $f(x) = \dfrac{x^2 - 3}{2x - 4}.$

26. Find $\lim\limits_{x \to 4} \dfrac{x + 1}{3x - 7}.$

27. If b, c and d are constants, for what value of b will be the curve $y = x^3 + bx^2 + cx + d$ have a point of inflection at $x = 1$?

28. Suppose that $f(-1) = 3$ and that $f'(x) = 0$ for all $x \in R$. Must $f(x) = 3$? Give reasons for your answer.

29. Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \leq x < 3$ is increasing and decreasing. Where does the function assume extreme values and what are these values ?

30. Find the area of the region enclosed by $x = 2y^2$, $x = 0$ and $y = 3$.

31. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

32. Find the length of the curve $= \dfrac{y^3}{} + \dfrac{1}{4y}$ from $y = 1$ to $y =$

33. Show that center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.

(6 x 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any two questions.
Each question carries 10 *marks.*

34. Find the center of mass of a thin plate of constant density 5 covering the legion bounded above by parabola y = 4 — x^2 and below by x-axis.

35. A spring has a natural length of 1 m. A force of 24 N stretches the spring to a length of 1.8 m.

    (a)   Find the force constant k.

    (b)   How much work will it take to stretch the spring 2 m. beyond its natural length ?

    (c)   How far will a 45 N force stretch the spring ?

36. What values of a and b make f (x) = x^3 + ax^2 + bx have,

    (a)   A local maximum at $x = -1$ and a local minimum at x = 3.

    (b)   A local minimum at x = 4 and a point of inflection at $x = 1$?

(2 x 10 = 20 marks)