D 71655 (Pages : 4) Name.................................

Reg. No.................................

THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Find $\lim_{x \to 3} \frac{x+3}{x^2-9}$.

2. Find $\frac{d}{dx} \ln(x^2+3)$.

3. Find $\lim_{x \to 0} \frac{3x - \sin x}{x}$.

4. Give an example of a sequence which has no upper bound.

5. Find a formula for the n^{th} term of the sequence $1, -4, 9, -16, 25, \ldots$

6. Find $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots$

7. Write a parametrization of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

8. $\lim_{n \to \infty} \frac{\ln n}{n} = \ldots$

Turn over

9. If $\sum_{n=1}^{\infty} a_n$ converges, then a_n converges to ...

10. Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$. If $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$, and $\sum b_n$ converges, then $\sum a_n$...

11. A series $\sum a_n$ is said to be absolutely convergent if ....

12. $\frac{d}{dx} a^x = \ldots$

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find $\lim_{x \to 0^+} x \cot x$.

14. Evaluate $\int_0^1 \sinh^2 x dx$.

15. Find $\lim_{n \to \infty} \frac{\cos n}{n}$.

16. Let $a_n = \begin{cases} \frac{n}{2n}, & n \text{ odd} ; \\ \frac{1}{2^n}, & n \text{ even.} \end{cases}$ Does Σa_n converge ?

17. For what values of x do the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converges ?

18. Find the center and radius of the conic section $x^2 + 4x + y^2 = 12$.

19. Locate the vertices of an ellipse of eccentricity 0.8 whose foci lie at the points $(0, \pm 7)$.

20. Determine the conic section from the equation $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$.

21. Graph the sets of points whose polar co-ordinates satisfy the condition $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$.

22. Find the polar equation for the circle $x^2 + (y-3)^2 = 9$.

23. Find the directrix of the parabola $r = \frac{5}{2 + 2\cos\theta}$.

24. Determine if the sequence $a_n = \frac{3n+1}{n+1}$ is non-decreasing and if it is bounded from above.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Show that $\lim_{x \to 0^+} (1+x)^{1/x} = e$.

26. Does the sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converge ? If so, find $\lim_{n \to \infty} a_n$.

27. Find a formula for the n^{th} partial sum of the series $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots + \frac{2}{3^{n-1}}$ and ude it to find the serie's sum if the series converges.

28. Find the surface area generated by revolving the curves $x = t + \sqrt{2}, y = \frac{t^2}{2} + \sqrt{2}t, -\sqrt{2} \leq t \leq \sqrt{2}$ about y-axis.

29. Show that the point $(2, 3\pi/4)$ lies on the curve $r = 2 \sin 2\theta$.

30. Find the Maclaurin series for the function $f(x) = xe^x$.

31. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges or diverges.

32. Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converges ?

33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

35. Find the Taylor series generated by $f(x) = 1/x$ at $a = 2$. Where if anywhere, does the series converges to $1/x$?

36. Find the length of the curve $x = t^2/2, y = \frac{(2t+1)^{3/2}}{3}, 1 \le t \le 4$.

(2 × 10 = 20 marks)