D 70323 (Pages : 4) Name.......................................

Reg. No..................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. The inverse Laplace transform of the function $f(t) = 1$ is ————.

2. The integrating factor fot the linear differential equation $y' - \frac{1}{t}y = 0$ is ————.

3. Write the order of the differential equation :

$$\frac{d^3y}{dx^3} + 4\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + 2y^2 = 0.$$

4. Show that the differential equation :

$$(2xy + y - \tan y) + \left(x^2 - x\tan^2 y + \sec^2 y + 2\right)y' = 0 \text{ is exact.}$$

5. Solve $y'' + y = 0$.

6. Are the functions $e^{\pi t}$ and $\frac{1}{\pi}e^{\pi t}$ linearly independent ?

7. Find the Laplace Transform of $e^{at}\cos bt$.

8. Define step function.

9. If $f(x)$ is an even function, the co-efficient of sines in the Fourier series expansion of $f(x)$ is evaluated by the integral ————.

Turn over

10. What is the fundamental period of $\cos\left(\frac{\pi x}{3}\right)$.

11. If $f(x) = x + k$ is an odd function, the value of k must be ————.

12. What is the heat conduction equation ?

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Show that $u(x, y) = \cos x \cosh y$ is a solution of the partial differential equation $u_{xx} + u_{yy} = 0$.

14. Find the solution of the initial value problem $y' = (1 - 2x)y^2, y(0) = \frac{1}{6}$.

15. Find the value of b for which the following equation is exact :

$$\left(xy^2 + b\,x^2\,y\right)dx + (x + y)\,x^2\,dy = 0.$$

16. Write the conditions for the existence of the Laplacè transform of a function.

17. Find the Wronskian of the functions x and xe^x.

18. Find the general solution of $y'' + 2y' + 5y = 0$.

19. Find the Laplace transform of $f(t) = e^{\omega t}, t \geq 0$.

20. Find the Laplace transform of $f(t) = 5e^{-2t} - 3\sin 4t, t \geq 0$.

21. Find the inverse Laplace transform of the function $\frac{1}{s^2 - 4s + 5}$.

22. Show that sum of two even functions is even.

23. Assuming the required equations, prove that $L[f'(t)] = sL[f(t)] - f(0)$.

24. Find $L\left(e^{5t} \cos 3ht\right)$.

25. Find the inverse Laplace transform of the function $\ln \frac{s+a}{s+b}$.

26. Find a_0 for the periodic function :

$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}.$$

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Show that the differential equation $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$ homogeneous and hence solve.

28. Find an integrating factor for the equation $\left(3xy + y^2\right) + \left(x^2 + xy\right) y' = 0$ and then solve the equation.

29. Transform the equation $u'' + 2u' + 2u = 0$ into a system of first order equation.

30. State and prove Abel's Theorem.

31. Using the method of Laplace transform, solve :

$y'' - 3y' + 2y = 4e^{2t}$, $y(0) = -3$, $y'(0) = 5$.

32. State and prove the convolution theorem for Laplace transform.

33. Find the inverse transform of $F(s) = \frac{1 - e^{-2s}}{s^2}$.

Turn over

34. Find the solution of the initial value problem :

$$2y'' + y' + 2y = \delta(t-5)\,;\; y(0) = 0,\; y'(0) = 0.$$

35. Solve using the method of separation of variables :

$$\frac{\partial u}{\partial x} = a^2 \frac{\partial u}{\partial y},\; u(x,0) = 1,\; u(0,y) = -1.$$

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) Solve by method of variation of parameters :

$$y'' + y = \tan t,\; 0 < t < \frac{\pi}{2}.$$

(b) Find the general solution of $t^2 y'' - 4ty' + 6y = 0, t > 0$.

37. Let $f(x) = 1 - x^2$ if $-1 \le x \le 1$ and $f(x+2) = f(x)$. Then :

(a) Sketch the graph of the function f and state whether the function is even or odd.

(b) Find the Fourier series of f.

(c) Deduce that : $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots\ldots = \sum_{n=1}^{\infty} \dfrac{1}{n^2}$.

38. Derive the wave equation by stating the assumptions involved and find it's D'Alembert's solution.

(2 × 13 = 26 marks)