**D 70320** **(Pages : 4)** **Name..........................................**

**Reg. No......................................**

# FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

## Part A

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. Evaluate $\lim_{(x, y) \to (1, 3)} \frac{x+1}{4-y}$.

2. Find the domain and range of $z = \sqrt{1 - x^2 - y^2}$.

3. Find the gradient of $\phi(x, y, z) = x^2 + y^2 + z^2$.

4. Compute the divergence of $\vec{f} = xy\,\vec{i} + yz\,\vec{j} + xz\,\vec{k}$.

5. Define directional derivative of a function.

6. What do you mean by a conservative vector field ?

7. Give a very brief discription of linearization of a function of two variables.

8. Find $du$ if $u = e^{x^2 + y^2 + z^2}$.

9. Fill in the blanks : If $\vec{f}$ and $\vec{g}$ are differentiable vector point functions, then

   $\nabla \cdot (\vec{f} \times \vec{g}) =$ .............

10. State the tangential form of Green's theorem in the plane.

Turn over

11. Fill in the blanks : If $\vec{a}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $\nabla \times (\vec{a} \times \vec{r}) =$ ..........

12. State Stokes theorem mentioning all the assumptions involved in it explicitly.

(12 × 1 = 12 marks)

**Part B**

*Answer any* **ten** *questions.*
*Each question carries 4 marks.*

13. Find the vector normal to the surface $\phi(x, y, z) = x^2y - 2y^2z^3$ at $(1, -1, 2)$.

14. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

15. If $x^2 + y^2 + z^2 + ye^x z + z\cos y = 0$ then, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at the origin.

16. Prove that $\nabla(r^n) = nr^{n-2}\vec{r}$.

17. Find the total derivative of $u = xy + z$ with respect to $t$ if $x = \cos t$, $y = \sin t$ and $z = t$.

18. Compute the average value of the function $f(x, y) = x\cos(xy)$ over the rectangular region $0 \le x \le \pi$, $0 \le y \le 1$.

19. Linearize the function $f(x, y) = \sin(\pi xy^2)$ at $(1, 1)$.

20. Find the directional derivative of $f(x, y) = xe^y + \cos(xy)$ at $(2, 0)$ in the direction of $3\vec{i} - 4\vec{j}$.

21. Find the velocity and acceleration vectors of $r(t) = (3\cos t)\,i + (3\sin t)\,j + t^2\,k$.

22. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix

$x = \cos t, y = \sin t, z = t\ ;\ 0 \le t\ \ \pi/2$.

23. Test whether the vector $\vec{f} = \left(e^x \cos y + yz\right)\vec{i} + \left(xz - e^x \sin y\right)\vec{j} + (xy + z)\ \vec{k}$ is conservative or not.

24. If the sides and angles in a traingle vary in such a way that its circum-radius R remains a constant, then show that $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0.$

25. Verify whether the differential $ydx + xdy + 4dz$ is exact or not.

26. Show that $\vec{f} \times \vec{g}$ is solenoidal if $\vec{f}$ and $\vec{g}$ are irrotational.

(10 × 4 = 40 marks)

## Part C

*Answer any* **six** *questions.*
*Each question carries 7 marks.*

27. Evaluate $\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x}\, dydx.$

28. If $\vec{f}$ is a differentiable vector function of $t$, differentiable at least 3 times, prove that

$$\frac{d}{dt}\left[\vec{f}, \vec{f}', \vec{f}''\right] = \left[\vec{f}, \vec{f}', \vec{f}'''\right].$$

29. Find the work done by the force field $\vec{f} = z\,\vec{i} + x\vec{j} + y\,\vec{k}$ along the boundary of the curve $C : \vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} + 3t\,\vec{k}$ where $0 \le t \le 2\pi$.

30. Test the continuity of $f\,(x, y)$ defined by

$$f\,(x, y) = \frac{xy}{x^2 + y^2}, (x, y) \neq (0, 0) \text{ and } f\,(x, y) = 0,\ (x, y) = (0, 0).$$

31. Find the equation to the tangent plane and normal line to the surface $f\,(x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at the point $(1, 2, 4)$.

32. Evaluate the area enclosed by the Lemniscate $r^2 = 4\cos 2\theta$ using double integrals.

Turn over

33. Find the Local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

34. Evaluate the volume of the region bounded by $x^2 + y^2 = 4$, $y + z = 3$, $z = 0$.

35. Show that $\vec{f} = y \sin z\, \vec{i} + x \sin z\, \vec{j} + xy \cos z\, \vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 13 *marks.*

36. (a) State Gauss divergence theorem and use it to evaluate the outward flux of $\vec{f} = xy\, \vec{i} + yz\, \vec{j} + xz\, \vec{k}$ through the surface of the cube cut from the first octant by the planes $x = y = z = 1$.

(b) If S is a closed surface enclosing a volume V, then prove that $\int\int_S \vec{r}\,.n\,dS = 3V$.

37. (a) Evaluate the surface integral $\int\int_S \vec{f}\,.n\,dS$ where $\vec{f} = y\, \vec{i} + x\, \vec{j} + z^2\, \vec{k}$ over the cylindrical surface S given by $x^2 + y^2 = a^2$, $z = 0, z = h$.

(b) Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -2, 2)$.

38. (a) Find the value of $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x\,dx + \cos y \sin x\,dy + dz$.

(b) In what direction from the point $(2, 1, -1)$ the directional derivative of $\phi(x, y, z) = x^2 yz^3$ is maximum and find the magnitude of this maximum.

(2 × 13 = 26 marks)