D 70321 (Pages : 3) Name.......................................

Reg. No...................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 06—ABSTRACT ALGEBRA

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : The total number of commutative binary operations on a set of n elements is ————.
2. Fill in the blanks : The number of elements in the ring $M_2(\mathbb{Z}_3)$ is ————.
3. Fill in the blanks : The least value of n such that a group G of order n is non-abelian is ————.
4. Define a group.
5. Give an example of a finite integral domain.
6. Define skew fields.
7. Calculate the order of the permutation $\mu = (1)\,(1\ 2)\,(1\ 3)$ in S_4.
8. Solve : $-3x + 2 = 4$ in the group $< \mathbb{Z}_6, +_6 >$.
9. Show that the identity element in a group is unique.
10. How many left cosets are there for $p\mathbb{Z}$ in $\mathbb{Z}$ if p is a prime.
11. What is a Klein group ?
12. Give a group theoretic definition of greatest common divisor of two positive integers.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. If H is a finite non-empty subset of a group G, establish that H will be a subgroup if it is closed under the binary operation in G.
14. Show that a group is a finite group if it has finite number of subgroups.

Turn over

15. Show that every cyclic group is abelian.

16. Find all group homomorphism from $\mathbb{Z}$ into itself.

17. Let G be a group of order pq where p and q are primes. Show that every proper subgroup of G is cyclic.

18. Let S be a set and let f, g and h be functions mapping S into S. Prove that $f*(g*h)=(f*g)*h$ where the binary operation * is the function composition.

19. Is the union of two subgroups a subgroup ? Justify your claim.

20. Show that the coset multiplication given by $(a\mathrm{H})(b\mathrm{H}) = ab\mathrm{H}$ is a well defined operation when H is a normal subgroup of G.

21. Draw the subgroup diagram for $\mathbb{Z}_{18}$.

22. Show that any finite cyclic group of order n is isomorphic to $\mathbb{Z}_n$.

23. Find a group isomorphic to the Klein group other than the Klein group. Establish that it is so.

24. Give any necessary and sufficient condition for a ring R to have no zero divisors. Justify your claim.

25. Is Q, the set of rationals, the field of quotients for integers? Give reasons to establish your assertion or denial.

26. Show that factor group of a cyclic group is always abelian.

($10 \times 4 = 40$ marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Draw the group table for the dihedral group D_4. Is D_4 a cyclic group ? Justify your claim.

28. Define kernal of a group homomorphism and show that it is a normal subgroup of the domain of the homomorphism.

29. Define order of an element in any group G. Show that in a finite group G, order of any element divides order of G.

30. Show that every permutation σ of a finite set is a product of disjoint cycles.

31. Prove or disprove : Every finite integral domain is a field.

32. Define the alternate group A_n. Show that it is a normal subgroup and find the group isomorphic to S_n/A_n.

33. Let G be a finite group in which for each positive integer m, the number of solutions of $x^m = e$ is at most m. Then show that G is cyclic.

34. If $< R, + >$ is an abelian group, show that $< R, +, . >$ is a ring if $a.b$ is defined as 0 for all $a,b \in R$.

35. Prove in some detail that every field L containing an integral domain D contains the field of quotients of D.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) If $\phi : G \to G'$ is a group homomorphism then show that $\phi[H] \leq G'$ whenever $H \leq G$.

(b) Find the index of the subgroup generated by $\sigma = (1, 5, 3, 4)(2, 3)$ in S_5.

37. (a) Prove that the converse of the Lagranges theorem need not be true.

(b) Express $\sigma = (1\ 2\ 3)(1\ 3\ 4)^2 \in S_4$ as a product of disjoint cycles.

38. (a) Define rings and ring homomorphisms. Show that the ring of real numbers and complex numbers are not isomorphic.

(b) Find all the units in the ring $< Z_{18}, +_{18}, \times_{18} >$.

(2 × 13 = 26 marks)