D 70973 **(Pages : 3)** **Name..................................**

Reg. No...............................

THIRD SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2019

(CUCSS)

Mathematics

MT 3C 15—PDE AND INTEGRAL EQUATIONS

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Find the partial differential equation of all planes which are at a constant distance d form the origin.
2. Show that $(x-z)(y-z)=1$ is a singular integral of $z = px + qy - 2\sqrt{pq}$.
3. Determine the region for which the two equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible.
4. Find the complete integral of $9\left(p^2 z + q^2\right) = 4$.
5. Determine the characteristic curve for solving the equation $z_x - zz_y + z = 0$ for every y and $x > 0$ with the initial conditions $x_0 = 0,\ y_0 = s,\ z_0 = -2s,\ -\infty < s < \infty$.
6. What is the domain of dependence for a point ?
7. State Cauchy's problem for first order equations.
8. Show that the solution of the Neumann problem is unique up to the addition of constant.
9. State the heat conduction problem in a plate with Neumann boundary.
10. Show that the function $y(x) = \left(1 + x^2\right)^{-\frac{3}{2}}$ is solution of the Volterra integral equation $y(x) = \dfrac{1}{1+x^2} - \displaystyle\int_0^x \frac{\xi}{1+x^2} y(\xi)\, d\xi$.
11. Define Fredholm integral equation of second kind and give an example for it.
12. Determine the characteristic value λ for the equation $y(x) = \lambda \displaystyle\int_0^{2\pi} \sin(x+\xi)\, y(\xi)\, d\xi$.
13. Show that the Kernel $k(x,\xi) = (3x-2)\xi$ has no characteristic number associated with (0, 1).
14. Determine the resolvent kernel associated with $k(x,\xi) = \sin(x+\xi)$ in $(0, 2\pi)$, in the form of a power series in λ.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. Find the general integral of the equation $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$.

16. Using Charpit's method to find two complete integral of a first order partial differential equation $pq = px + qy$.

17. Solve the equation $p^2 x + q^2 y = z$ by Jacobi's method.

18. Reduce the equation $u_{xx} = (1+y)^2 u_{yy}$ into Canonical form.

19. Show that $\upsilon(x, y\ ;\ \alpha, \beta) = \dfrac{(x+y)[2xy + (\alpha - \beta)(x-y) + 2\alpha\beta]}{(\alpha + \beta)^3}$ is the Riemann function for the second order partial differential equation.

20. Show that the surfaces $x^2 + y^2 + z^2 = c^{\frac{2}{3}}$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.

21. Solve :

$$u_t = u_{xx}, 0 < x < l, t > 0$$
$$u(0,t) = u(l,t) = 0$$
$$u(x,0) = \begin{cases} x, & 0 \le x \le \frac{l}{2} \\ l - x, & \frac{l}{2} \le x \le l. \end{cases}$$

22. Transform the boundary value problem $\dfrac{d^2y}{dx^2} + \lambda y = 0,\ y(0) = 0,\ y(l) = 0$ to an integral equation.

23. Show that the characteristic function of the symmetric kernel corresponding to distinct characteristic numbers are orthogonal.

24. Solve by iterative method : $y(x) = \lambda \int_0^1 e^{x-\xi} y(\xi)\, d\xi + f(x)$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. Show that the Paffian differential equation $\left(y^2 + yz\right)dx + \left(xz + z^2\right)dy + \left(y^2 - xy\right)dz = 0$ is integrable and find the corresponding integral.

26. Determine the characteristic of the equation $pq = z$ which passes through the parabola $x = 0$, $y^2 = z$.

27. Show that the solution of the Dirichlet problem for a circle of radius a is given by Poisson integral formula.

28. Show that any solution of the integral equation

$$y(x) = \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi + F(x)$$

can be expressed as the sum of $F(x)$ and some linear combination of the characteristic functions.

($2 \times 4 = 8$ weightage)