D 72972 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION DECEMBER 2019

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Define a graph isomorphism. Illustrate with an example.

2. Prove or disprove : If G is connected and E' isthe set of edges whose deletion results in a disconnected graph then E' contains an edge cut of G.

3. Show that every Eulerian graph is connected.

4. Prove that a connected graph is a tree if and only if it has only one face.

5. Define partial order and total order relations on a set X. Give an example of a partial order which is not a total order.

6. If $(X, +, \cdot, ')$ is a Boolean algebra, prove that :

   (i) $(x + y)' = x' \cdot y'$; and (ii) $(x \cdot y)' = x' + y', x, y \in X$.

7. Find a grammar that generates $L = \{a^n b^{n+1} : n \geq 0\}$.

8. Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer **six** *questions, choosing* **two** *from each unit.*
Each question carries weightage 2.

Unit I

9. (a) If G is a simple graph with $\delta \geq \frac{n-1}{2}$, prove that G is connected.

   (b) If the simple graph G is not connected, then show that G^c is connected.

10. (a) If G is a simple connected graph with a cut edge and $n(G) \geq 3$, prove that G has a cut vertex.

    (b) Show that a simple graph G with n vertices, $n \geq 2$, is complete if and only if $k(G) = n - 1$.

11. (a) Prove that a graph is planar if and only if it is embeddable on a sphere.

    (b) If G is a simple connected planar bipartite graph with $n \geq 3$, then prove that $m \leq 2n - 4$.

Unit II

12. Let $(X, \leq)$ be a partially ordered set and $x \in X$. Let $A = \{x \in X : x < z\}$. Prove that $y \in X$ covers x if and only if y is a minimal element of A.

13. (a) What is a Boolean algebra. Give a non-trivial example of a Boolean algebra with two elements.

    (b) What is a subalgebra of the Boolean algebra $(X, +, \cdot, ')$. If Y is a subalgebra of the Boolean algebra $(X, +, \cdot, ')$, prove that $(Y, +, \cdot, ')$ is also a Boolean algebra.

14. (a) What is a Boolean variable and a Boolean function.

    (b) Write the Boolean function $f(a, b, c) = (a + b + c)(a' + b + c')(a + b' + c')(a' + b' + c')(a + b + c')$ in its DNF.

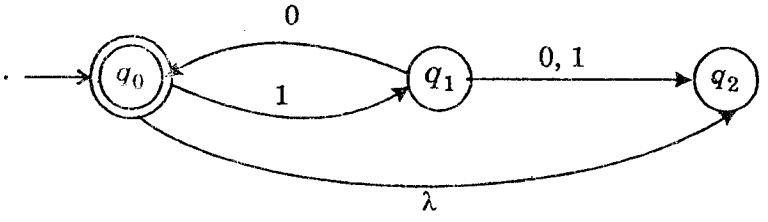
Unit III

15. (a) Find dfa for $L = \{w : |w| \bmod 3 = 0\}$, with $\sum = \{a, b\}$.

    (b) Find a nfa with three states that accepts the language $\{ab, abc\}^*$.

16. Find a dfa that accepts the set of all strings on $\sum = \{a, b\}$ starting with the prefix ab.

17. Find the language accepted by the automaton in the following transition diagram :



(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 5.

18. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.

(b) For any loopless connected graph G, prove that k (G) $\leq$ λ (G) $\leq$ δ (G).

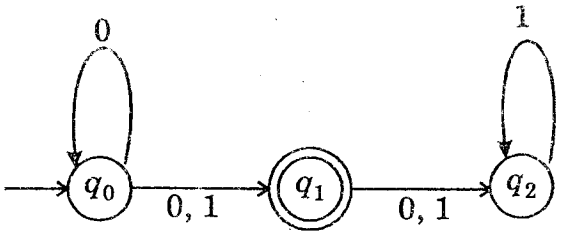
19. (a) If each block of G is planar, prove that the graph G itself is planar.

(b) Show that the complement of a simple planar graph with 11 vertices is non-planar.

20. (a) Prove that a symmetric Boolean function is completely determined by its characteristic numbers.

(b) Show that every Boolean function on n variables $x_1, x_2, \ldots, x_n$ can be uniquely expressed as the sum of terms of the form $x_1^{\epsilon_1} x_2^{\epsilon_1} \ldots x_n^{\epsilon_n}$ where each $x_i^{\epsilon_i}$ is either x_i or x_i'.

21. Convert the nfa given by the following transition diagram into an equivalent dfa.



(2 × 5 = 10 weightage)