C 81805 (Pages : 4) Name..................................

Reg. No..................................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2020

Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the minimum value of $f(x) = x^2 - 1$ on $[-1, 2]$.

2. Find the critical points of $f(x)$ if $f'(x) = (x-1)(x+2)(x-3)$.

3. Find $\lim\limits_{x \to \infty} \dfrac{2x+3}{5x+7}$.

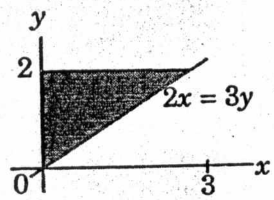
4. Write the sum $\sum\limits_{k=-1}^{1} \dfrac{(-1)^k}{k+2}$ without sigma notation.

5. Evaluate $\int\limits_{-3}^{0} [-g(x)]\,dx$ if $\int\limits_{-3}^{0} g(t)\,dt = \sqrt{2}$.

6. State Fundamental Theorem of Calculus.

7. Evaluate $\int\limits_{0}^{1} \left(x^2 + \sqrt{x}\right) dx$.

8. Set up an integral to find the volume of the solid generated by revolving the shaded region about the y-axis :



Turn over

9. Set up an integral to find the area of the shaded region of the figure in Question 8.

10. Set up an integral to find the length of the curve $y = x^{3/2}, 0 \le x \le 4$.

11. Set up an integral to find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le 1/2$ about the x-axis.

12. Find the work done by a force $F(x) = x^2$ N along the x-axis from $x = 1$ m to $x = 3$ m.

(12 × 1 = 12 marks)

Part B

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Find the absolute maximum value of $f(x) = x^{4/3}$ on [– 1, 8].

14. If $f'(x) = 0$ at each point of an interval I, prove that $f(x) = c$ for all x in I, where c is a constant.

15. Find $f(2)$, if $f(1) = 0$ and $f'(x) = 2x$ for all x.

16. Evaluate $\sum_{k=1}^{5} k(3k+5)$.

17. State Rolle's Theorem.

18. Find $\frac{dy}{dx}$ if $y = \int_{1}^{x^4} \sqrt{u}du$.

19. Evaluate $\int_{0}^{1} t^3 \left(1+t^4\right)^3 dt$.

20. Find the area of the region in the first quadrant enclosed by the curves $x = y^2$ and $x = y^3$.

21. A pyramid 3 m. high has a square base that is 3 m. on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

22. Find the volume of the solid generated by revolving the region bounded by the curve

$x = \sqrt{2 \sin 2y}, 0 \le y \le \pi/2$ and the line $x = 0$ about the y-axis.

23. Find the length of the curve $y^2 + 2y = 2x + 1$, from $(-1, -1)$ to $(7, 3)$.

24. Find the work required to compress a spring from its natural length of 0.75 ft if the force constant is $k = 16$ lb/ft.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. State and prove Mean Value Theorem.

26. Find the asymptotes of the curve $y = 2 + \frac{\sin x}{x}$ using Sandwich Theorem.

27. Find the value of c in the Mean Value Theorem for $f(x) = x^2$ in $[0, 2]$.

28. What is the smallest perimeter possible for a rectangle whose area is 16 in^2.

29. Find the linearization of $f(x) = x^3 - x$ at $x = 1$.

30. Find the area of the region enclosed by the curves $y = \frac{x^2}{4}$ and the lines $y = x, y = 1$.

31. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

32. Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.

33. Find the center of mass of a wire of constant density δ shaped like a semicircle of radius a.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (i) Find the intervals on which $h(x) = -x^3 + 2x^2$ is increasing and decreasing. Identity the local extreme values, if any, of $h(x)$, saying where they are taken on. Which of the extreme values are absolute ?

    (ii) Graph the function $y = \dfrac{x^3 - 1}{x}$.

35. (i) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \le x \le \frac{1}{2}$, about the x-axis.

    (ii) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the initial line $y = x - 2$.

36. (i) Find the center of mass of a thin plate of constant density δ covering the region bounded by the parabola $y = 4 - x^2$ and below by the x-axis.

    (ii) A spring has a natural length of 1 m. A force of 24 N stretches the spring to a length of 1.8 m.

    (a) Find the force constant k.

    (b) How much work will it take to stretch the spring 2 m. beyond its natural length ?

    (c) How far will a 45-N force stretch the spring ?

$(2 \times 10 = 20$ marks)