C 81823 maths (Pages : 4) Name..............................

Reg. No..............................

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**

Each question carries 1 *mark.*

Name the following :

1. $E[X-E(X)]^r$ of a random variable X.

2. For two random variables X and Y, $E[(x-\bar{x})(y-\bar{y})]$.

3. If $X \sim N(0,1)$, then the distribution of the square of X.

Fill up the blanks :

4. X and Y are independent random variables with

   $f_1(x) = e^{-x}, x > 0$ and $f_2(x) = e^{-y}, y > 0$, then $f(x/y) =$ ————————.

5. The mode of a Poisson random variable with $E(X) = 5$ is ————————.

6. X is the number turns up when a fair die is tossed, X follows ———————— distribution.

7. If X follow exponential distribution with mean 0.5, then $P(X > 4) =$ ————————.

Write True *or* False :

8. Fourth cumulant k_4 of a random variable X is μ_4.

9. The mean and standard deviation of X following Poisson distribution are same.

10. Tchebycheve's inequality exists only for the random variable with finite variance.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in* **one sentence.**
Each question carries 2 marks.

11. For two random variables, prove that $\text{Cov}(X+Y, X-Y) = V(X) - V(Y)$.

12. Find $E(X)$, X denotes the square of the number shown by a fair coin when it is tossed.

13. Define Bernoulli distribution.

14. $Y \sim N(1,1)$, find (i) $P(Y>0)$; (ii) $P(Y<2)$.

15. If X follow rectangular distribution over [2, 6], find the mean of X.

16. Define Cauchy distribution.

17. State Central Limit theorem.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each one carries 4 marks.

18. State and prove the addition theorem on expectation for two random variables X and Y.

19. Given the joint p.m.f. of X and Y, $f(x, y) = \frac{x+2y}{18}, x = 1, 2; y = 1, 2.$ Find $E(X^2Y)$.

20. For two random variables X and Y, prove that $E(E(X/Y)) = E(X)$.

21. For a random variable X and for two constants 'a' and 'b', prove that $M_{aX+b}(t) = e^{bt} M_X(at)$.

22. If X follows follow discrete uniform distribution over {1, 2, 3...n}, find V (X).

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each one carries 6 *marks.*

23. For two random variables X and Y, the joint p.m.f. $f(x, y) = \frac{x^2 + 2y}{22}, x = 1, 2; y = 1, 2$. Find the condition probability distributions of (i) X/Y = 1 ; and (ii) Y/X = 2.

24. Prove that $-1 \leq r_{xy} \leq 1$, where r_{xy} is Pearson's co-efficient of correlation between any *two* random variables X and Y.

25. State and prove the lack of memory property of geometric distribution.

26. Prove that $\mu_{r+1} = r\lambda\mu_{r-1} + \lambda\frac{d}{d\lambda}\mu_r$, where $\mu_{r-1}, \mu_r, \mu_{r+1}$ are the central moments of Poisson distribution with parameter λ.

27. If X is exponential random variable with $f(x) = \lambda e^{-\lambda x}, x > 0$, show that $Y = 1 - e^{-\lambda x}$ follow rectangular distribution over [0, 1].

28. Define convergence in probability and convergence in distribution. If X denotes the total number of successes in n Bernoulli trials with probability of success p, prove that $\frac{X}{n}$ converges to p in probability.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each one carries 10 *marks.*

29. Let X and Y are two random variables with joint pdf $f(x, y) = 8xy,\ 0 < x < y < 1$ and $f(x, y) = 0$, elsewhere. Find Correlation between X and Y.

30. (i) Define binomial distribution.

    (ii) If X following binomial distribution with parameters n and p, (a) obtain the m.g.f. of X ; (b) show that $\text{Cov}\left(\frac{X}{n}, \frac{n-X}{n}\right) = \frac{-pq}{n}$.

Turn over

31. For a random variable following $N(\mu, \sigma)$, show that (i) $E(X) = \mu$; (ii) $V(X) = \sigma^2$; and (iii) $M_x(t) = e^{\mu t + \frac{t^2\sigma^2}{2}}$.

32. State and prove weak law of large numbers. Also prove that a sequence of independent and identically distributed random variables $\{X_n\}$ obeys weak law of large numbers.

(2 × 10 = 20 marks)