C 82445 (Pages : 3) Name..............................

Reg. No..............................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—I

(2019 Admissions)

Time : Two Hours and a Half | Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. Let f and g be functions defined by $f(x) = x + 1$ and $g(x) = \sqrt{x}$. Find the functions gof and fog. What is the domain of gof?

2. Find $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}$.

3. Let $f(x)=\begin{cases}\dfrac{x^2-x-2}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2.\end{cases}$ Show that f has a removable discontinuity at 2. Redefine f at 2 so that it is continuous everywhere.

4. Find $\lim\limits_{x\to\pi/4}\dfrac{\sin x}{x}$.

5. Show that $f(x) = |x|$ is continuous everywhere.

6. Find the derivative of $\sqrt[3]{x}+\dfrac{1}{\sqrt{x}}$.

7. Find the critical points of $f(x) = x^3 - 6x + 2$.

8. Find $\lim\limits_{x\to\infty}\left(2x^3 - x^2 + 1\right)$ and $\lim\limits_{x\to-\infty} 2x^3 - x^2 + 1$.

9. Find the interval on which $f(x) = x^2 - 2x$ is increasing or decreasing.

10. Find the vertical asymptote of the graph of $f(x) = \dfrac{1}{x-1}$.

Turn over

11. Find $\int \frac{\cos x}{1-\cos^2 x}\,dx$.

12. Find $\int xe^{-x^2}\,dx$.

13. Suppose $\int_1^6 f(x)dx = 8$ and $\int_4^6 f(x)dx = 5$, what is $\int_1^4 f(x)dx$.

14. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on [0, 2] about the x-axis.

15. Find the work done in lifting a 2.4 kg. package 0.8 m. off the ground (given g = 9.8 m./sec.^{2}).

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 marks.

16. Find the slope and an equation of the tangent line to the graph of the equation $y = -x^2 + 4x$ at the point p (2, 4).

17. Suppose that $g(x) = (x^2+1)f(x)$ and it is known that $f(2) = 3$ and $f'(2) = -1$. Evaluate $g'(2)$.

18. (a) Show that $f(x) = x^3$ satisfies the hypothesis of the mean value theorem on [– 1, 1].
    (b) Find the numbers c in (– 1, 1) that satisfies the equation as guaranteed by the mean value theorem.

19. Find the slant asymptotes of the graph of $f(x) = \frac{2x^2-3}{x-2}$.

20. A car moves along a straight road with velocity function $v(t) = t^2 + t - 6, 0 \le t \le 10$, where $v(t)$ is measured in feet per second.
    (a) Find the displacement of the car between t = 1 and t = 4.
    (b) Find the distance covered by the car during this period.

21. (a) Evaluate $\int_{-3}^{0}(x^2-4x+7)\,dx$ by Fundamental theorem of Calculus.
    (b) Use the definition of definite integral to show that if $f(x) = c$, a constant function, then $\int_a^b f(x)\,dx = c(b-a)$.

22. Find the center of mass of a system comprising three particles with masses 2, 3 and 5 slugs, located at the points (– 2, 2), (4, 6) and (2, – 3) respectively.

23. Find the lengh of the graph of $x = \frac{1}{3}y^3 + \frac{1}{4y}$ from $P\left(\frac{7}{12},1\right)$ to $G\left(\frac{67}{24},2\right)$.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}$.

(b) Use intermediate value theorem to find the value of c such that $f(c) = 7$, where $f(x) = x^2 - x + 1$ on $[-1, 4]$.

(c) In a fire works display, a shell is launched vertically upwards from the ground, reaching a height $S = -16t^2 + 256t$ feets after t seconds. The shell burst when it reaches its maximum height :

(i) A what time after launch will the shell burst.

(ii) What will be the altitude of the shell when it explodes ?

25. Find the dimensions of the rectangle of greatest area that has its base on the x-axis and is inscribed in the parabola $y = 9 - x^2$.

26. Using the definiton of definite integral evaluate $\int_a^b x\,dx$.

27. Find the aera of the surface obtained by revolving the graph of $f(x) = \sqrt{x}$ on the interval $[0, 2]$ about the x-axis.

(2 × 10 = 20 marks)