C 82459 (Pages : 2) Name.....................................

Reg. No.....................................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Statistics

STA 2C 02—PROBABILITY THEORY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. Define sample space, event of a random experiment.
2. Explain mutually exclusive and exhaustive events.
3. If $P(A) = 0.6$, $P(A \cup B) = 0.8$, find $P(B)$ when A and B are independent.
4. Define $P(A/B)$, where A and B are two events. Also state the multiplication theorem on probability.
5. Differentiate discrete and continuous random variables.
6. Find k, if $f(x) = kx^2$, for $0 < x < 1$ is a probability density function of X.
7. For a random variable X with possible values 1, 2 and 3, identify with reason, the values F(0.5) and F(3.2) where F is the distribution function of X.
8. Define Mathematical expectation of a discrete random variable X. Also show that, for a random variable X, $[E(X)]^2 \le E(X^2)$ if the expectations exist.
9. If $M_X(t)$ is the m.g.f. of X, identify the m.g.f. of $2X - 5$.
10. Find the characteristic function of X, where $P(X = x) = 0.5$; for $x = 0.1$.
11. Express coefficient of correlation between two random variables X and Y in terms of expectations.
12. If the joint p.d.f. of X and Y is $f(x, y) = 1$, for $0 < x < 1; 0 < y < 1$, find $P(X > 0.2/Y > 0.6)$.

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. For two events A and B, $P(A) = 0.4$, $P(B) = 0.3$, $P(A \cap B) = 0.2$. Find (i) P(At least one of A and B to happen) ; (ii) P(Exactly one of A and B to happen).
14. For two events A and B, prove that $P(A \cup B)/C = P(A/C) + P(B/C) - P(A \cap B/C)$.

Turn over

15. Identify the distribution function of X and sketch its graph when the possible values of X are $-1, 0, 1$ and 2 with respective probabilities 0.2, 0.35, 0.4 and 0.05.

16. Given the p.d.f. of X as $f(x) = 1$, for $0 < x < 1$. Find the p.d.f. of $Y = -\log_e X$.

17. Given the p.d.f. of X as $f(x) = e^{-x}$, for $0 < x < \infty$. Find the m.g.f. of X and hence the variance of X using m.g.f.

18. The first three raw moments of X are $\lambda, \lambda^2 + \lambda$ and $\lambda^3 + 3\lambda^2 + \lambda$. Obtain the coefficient of skewness of X and identify the condition for symmetry.

19. State and prove Cauchy-Schwartz inequality for two random variables X and Y.

Part C (Essay Type Questions)

*Answer any **one** question.*
The question carries 10 marks.
Maximum marks that can be scored from this part is 10.

20. State and prove Bayes' theorem. Result of a survey from a college consists of 40 % boys and 60% girls on a recently released film, reveals that 35 % of the boys like the film but 30 % of the girls not like the film. A randomly selected student from this college likes the film. What is the probability that the student is a girl ?

21. (a) State and prove the multiplication theorem on expectation for the two random variables X and Y.

    (b) If the joint p.d.f. of (X, Y) is $f(x, y) = cxy$, for $0 < x < y < 1$.

    (i) Find the value of c ; (ii) Verify whether X and Y are independent.