D 90233 | 18/3 FN | (Pages : 4) | Name..
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FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCBCSS–UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours | Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : The solution of $y' = 2y, y(0) = 1$ is $y(t) =$ ————.
2. Write the wave equation and the conditions assumed for its D'Alemberts solution.
3. Find the general solution of $y'' - 2y = 0$.
4. Find the Laplace transform of cosh ($2\,at$).
5. Write the Euler formulas for computing the Fourier coefficients in the Fourier series expansion of a periodic function $f(x)$ of period 2L.
6. Define an exact differential equation. Is $(2x+y)dy - (x-y)dx = 0$ exact ? Why ?
7. Solve the system : $\dfrac{dx}{dt} = 2y, \dfrac{dy}{dt} = -2x.$
8. Define Dirac's delta function and write its Laplace transform.
9. Give an example of a non-linear differential equation in the dependent variable y and the independent variable t of second order.
10. Show that $u(x,y) = f(x-ay) + g(x+ay)$ is a solution of the partial differential equation
$$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial y^2}.$$
11. Show that product of two odd functions is an even function.
12. Compute the Wronskian of the functions e^{3t} and e^{-3t}.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 48.

13. $\left(x+e^{-y/3}\right)\dfrac{dy}{dx}=3, y(0)=0.$

14. Use Laplace transform to find the solution of $\dfrac{dy}{dt}=t, y(0)=1.$

15. Using convolution find the inverse Laplace transform of $\dfrac{1}{(s-2)(s-1)}$.

16. Show that any separable equation $M(x)+N(y)y'=0$ is also exact.

17. Solve : $t^2y''+ty'+y=0.$

18. Use method of variation of parameters to solve : $y''+4y=3\csc t$.

19. Compute the Wronskian of vectors $x^{(1)}(t)=\begin{pmatrix} t \\ 1 \end{pmatrix}$ and $x^{(2)}(t)=\begin{pmatrix} t^2 \\ 2t \end{pmatrix}$.

20. Find an integrating factor and solve it : $y'=e^{2x}+y-1.$

21. Find the formula for computing $\mathcal{L}\left(t^n\right)$ when n is an integer in terms of the gamma function.

22. Find the value of a_n, the Fourier coefficient of $\cos nx$ in the Fourier expansion of $f(x)=k$ for $x\in[-\pi, \pi]$.

23. State Abels theorem.

24. Solve : $\left(3xy+y^2\right)+\left(x^2+xy\right)y'=0.$

25. Briefly describe about mathematical modelling by giving a physical example.

26. Find the longest interval in which the solution of the initial value problem :

$\left(t^2-3t\right)y''+ty'-(t+3)y=0, y(1)=2, y'(1)=1$ is certain to exist.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 marks.
All questions can be attended.
Overall Ceiling **45.**

27. Determine whether the equation is exact or not and hence find its solution :

$$\left(3x^2 - 2xy + 2\right)dx + \left(6y^2x^2 + 3\right)dy = 0.$$

28. Find an integrating factor for the equation $\left(3xy + y^2\right) + \left(x^2 + xy\right)y' = 0$ and solve it.

29. Find the general solution of $y'' - 2y' + y = 2\cos(2t) - t^2$.

30. Find the Fourier cosine series expansion of $f(t) = |t| + t \in [0, \pi]$.

31. Find (a) $\mathcal{L}\left(2te^{-2t} - t^2\cos(2t)\right)$ and (b)$\mathcal{L}^{-1}\left[\frac{\left(1-e^{-s}\right)}{s}\right]$.

32. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove that it is so.

33. Find the solution of the heat conduction problem :

$$100u_{xx} = u_t, 0 < x < 1,\ t > 0;\ u(0,t) = 0,\ u(1,t) = 0, t > 0;\ u(x,0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

34. (a) Show that $W\left(e^{\beta t}\cos\alpha t, e^{\beta t}\sin\alpha t\right) = \alpha e^{2\beta t}$ and (b) write the differential equation of second order for which $e^{\beta t}\cos\alpha t$ and $e^{\beta t}\sin\alpha t$ form a fundamental set of solutions.

35. A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m. high. Neglect air resistance :

    (a) Find the maximum height above the ground that the ball reaches.

    (b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

(5 × 9 = 45 marks)

Section D

*Answer any **one** question.*
Each question carries 15 marks.

36. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

37. Find the solution of the initial value problem $y'' - 2y - 1 = 0$, $y(0) = 0$, $y'(0) = 1$ in two ways ; one of them must be using Laplace transforms.

38. Find the two half range expansions of the function :

$$f(x) = \begin{cases} \dfrac{2kx}{L} & \text{, if } 0 < x < L/2 \\ \dfrac{2k(L-x)}{L} & \text{, if } L/2 < x < L. \end{cases}$$

(1 × 15 = 15 marks)