D 90230 15/03 F-N Maths **(Pages : 4)** Name.................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the domain and range of $u(x, y) = \sin xy$.

2. Evaluate $\underset{(x,y)\to(1,2)}{\text{Lt}} \left[\dfrac{3 - x + y}{4 + x - 2y}\right]$.

3. If $\vec{a}$ is a constant vector, $\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\vec{\nabla}(\vec{r}\cdot\vec{a}) =$ ———.

4. When do we say a vector is Solenoidal ?

5. What do you mean by irrotational vector ?

6. Find the total differential of $\ln(x\,y\,z)$.

7. What is the linearization of the function $f(x, y, z)$ at the point (x_0, y_0, z_0) ?

8. Write the condition for the differential form $P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz$ to be exact ?

9. State the normal form of Green's theorem in the plane.

10. If $\vec{r} = x\hat{i} + y\hat{i} + z\hat{k}$, and $r = |\vec{r}|$, then $r^n\,\vec{r}$ is solenoidal if $n =$ ———.

Turn over

11. State Gauss Divergence theorem.

12. If $\vec{f}$ and $\vec{g}$ are irrotational vector functions then $\operatorname{div}(\vec{f} \times \vec{g}) =$ ________.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Evaluate $\lim\limits_{(x, y) \to (0, 0)} \left[\dfrac{x - y}{x + y}\right]$.

14. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at the *pt* (π, π, π) from $\sin(x + y) + \sin(y + z) + \sin(z + x) = 0$.

15. Find the outward unit normal vector to the surface $\varphi(x, y, z) = 3x^2 y - y^3 z^2$ at $(1, -2, -1)$.

16. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.

17. Find the total derivative of $u(x, y, z) = xyz$ with respect to t where $x = t + 1, y = t^2 + 1, z = t^3 + 1$.

18. Find the linearization of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point $(2, 1, 0)$?

19. Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at the *pt* $(1, 2, 3)$ in the direction of $3\hat{i} + 4\hat{j} + 5\hat{k}$.

20. Using double integrals, obtain the area of the Lemniscate $r^2 = 4 \cos 2\theta$.

21. Using triple integrals find the average value of $f(x, y, z) = xyz$ over the boundary of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

22. Evaluate $\int_1^2 \int_0^x \frac{dxdy}{x^2 + y^2}$.

23. Find the circulation of $\overline{\mathrm{F}} = (x - y)\,\hat{i} + x\hat{j}$ around the unit circle centered at the origin.

24. Show that the vector field $\overline{\mathrm{F}} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ is conservative.

25. It $g(x, y, z)$ has continuous 2nd order partial derivatives, show that $\overline{\nabla} g$ is irrotational.

26. State the tangential form of Green's theorem in the plane. Also state it's generalization in space.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 marks.
All questions can be attended.
Overall Ceiling 45.

27. Evaluate $\int_C \overline{\mathrm{F}} \cdot dr$, where $\overline{\mathrm{F}} = \left(x^2 + y^2\right)\hat{i} - 2xy\,\hat{j}$ where C is the rectangle in xy plane bounded by $x = 0, x = a, y = 0$ and $y = b$.

28. If $\vec{f}(t)$ is a differentiable vector function of a scalar variable t, then prove that :

$$\frac{d}{df}\left[\vec{f}\,\vec{f}', \vec{f}''\right] = \left[\vec{f}\,\vec{f}''\,\vec{f}'''\right].$$

29. Test the continuity of $f(x, y) = \left\{\begin{array}{ll} \frac{xy}{x^2 + y^2} & ; (x, y) = (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{array}\right\}$ at the origin.

30. Show that the work done by the Force field $\overline{\mathrm{F}} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is independent of the path joining the points $(-1, 3, 9)$ and $(1, 6, -4)$. Also find the work done along any smooth curve joining $(-1, 3, 9)$ and $(1, 6, -4)$.

Turn over

31. Find the local extreme values of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

32. Using triple integral find the volume of the ellipsoide $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

33. Evaluate the area of the region in the xy plane enclosed by $x^2 + y^2 = 4$, $y = 1$ *and* $y = \sqrt{3}\, x$.

34. Integrate the function $f(x, y, z) = xyz$ over the surface of the cube cut from the 1st octant by the planes $x = 1$, $y = 1$ and $z = 1$.

35. Evaluate $\displaystyle\int_{(1,0,0)}^{(0,1,1)} \sin y \cos x dx + \cos y \sin x dy + dz$.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. Verify Gauss Divergence theorem for $\vec{F} = x\,\hat{i} + xy\hat{j} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

37. (a) Verify the tangential form of Green's theorem for $\vec{F} = (x - y)\,\hat{i} + x\hat{j}$ and the Region R bounded by the unit circle $x^2 + y^2 = 1$.

(b) State the Fundamental theorem of line integrations.

38. (a) Evaluate $\displaystyle\int_C \vec{f} \cdot d\vec{r}$, if $\vec{f} = y^2\hat{i} + x^2\hat{j} - (x + z)\,\hat{k}$, C is the boundary of the triangle with vertices (0,0, 0), (1, 0, 0) and (1, 1, 0).

(b) For any closed surfaces enclosing of a volume V prove that $\displaystyle\iint_S \operatorname{curl} \vec{f} \cdot \hat{n}\, dS = 0$.

(1 × 15 = 15 marks)