C 1246 (Pages : 4)

Name.....................................

Reg. No.....................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2021

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Define absolute maximum of a function.
2. State Bolzano's Intermediate Value Theorem.
3. Give an example for a uniformly continuous function which is not a Lipschitz function.
4. Write an example for a Riemann integrable function.
5. State Boundeness theorem.
6. Find $\|P\|$ if $P = \{0, 1, 2, 4\}$ in a partition of $[0, 4]$.
7. Define pointwise convergence of a sequence of functions.
8. Define uniform norm of a bounded function ϕ on $A \subset \mathbb{R}$.
9. $\lim\limits_{n \to \infty} \frac{x}{n} =$
10. $\int_1^{\infty} \frac{1}{x^2}\, dx =$
11. Write an example for a conditionally convergent improper integral.
12. Define Gamma function.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*

Each question carries 6 *marks.*

All questions can be attended.

Overall Ceiling 48.

13. State and prove Bolzano's Intermediate Value theorem.

14. Let I be an interval and let $f: \mathrm{I} \to \mathbb{R}$ be continuous on I. Then prove that $f(\mathrm{I})$ is an interval.

15. Define Lipchitz function. If $f : \mathrm{A} \to \mathbb{R}$ is a Lipschitz function, prove that f is uniformly continuous on A.

16. Show that every constant function on $[a, b]$ is in $\mathfrak{R}[a, b]$.

17. Suppose that $f, g \in \mathfrak{R}[a, b]$. Prove that $f + g \in \mathfrak{R}[a, b]$.

18. Evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$.

19. Let (f_n) be a sequence of bounded functions on $\mathrm{A} \subseteq \mathbb{R}$. Suppose that (f_n) converges uniformly on A to f. Then prove that $\| f_n - f \|_{\mathrm{A}} \to 0$.

20. Show that $\lim\limits_{n \to \infty} \frac{x}{x+n} = 0$ for all $x \in \mathbb{R}, x \geq 0$.

21. State and prove Weierstrass M-Test for a series of functions.

22. Test the convergence of $\int_1^\infty \frac{1}{x}\, dx$.

23. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

24. Prove that $\Gamma(n+1) = n\,\Gamma(n)$ for $n > 0$.

25. Show that $\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$.

26. Evaluate $\int_0^{\infty} e^{-x^2} dx$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*

Each question carries 9 *marks.*

All questions can be attended.

Overall Ceiling 45.

27. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that f is uniformly continuous on I.

28. Let $I = [a, b]$ be a closed bounded interval and let $f : I \to \mathbb{R}$ be a continuous function on I. Then prove that f has an absolute maximum and absolute minimum on I.

29. State and prove Continuous extension theorem.

30. If $f \in \Re[a, b]$, then prove that f is bounded on $[a, b]$.

31. State and prove Squeeze Theorem.

32. State and prove Cauchy Criterion for uniform convergence of sequence of functions.

33. Show that $\int_{-\infty}^{\infty} \frac{1+x}{1+x^2} dx$ diverges.

34. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, \forall\, m, n > 0$.

35. Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of Gamma functions and evaluate the integral $\int_0^1 x^5 (1 - x^3)^{10} dx$.

(5 × 9 = 45 marks)

Turn over

Section D

Answer any **one** *question.*

The question carries 15 *marks.*

36. (a) State and prove Location of roots theorem.

    (b) Test the uniform continuity of $f(x) = x^2$ on $[0, 2]$.

37. (a) State and prove Cauchy Criterion for Riemann Integrability.

    (b) Show that Dirichlet function is not Riemann Integrable.

38. (a) State and prove First form of Fundamental Theorem of Calculus.

    (b) Show that $\lim \dfrac{nx}{1+n^2x^2} = 0$ for all $x \in \mathbb{R}$.

(1 × 15 = 15 marks)