251 (Pages : 4) Name..................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
MARCH 2021

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions. Each question carries* 1 *mark.*

1. What is meant by a convex polyhedron ?
2. Explain the convex combination of set of vectors.
3. Write down the standard form of a general LPP.
4. Define feasible and optimal solution of a linear programming problem.
5. State fundamental theorem of linear programming.
6. In the two-phase simplex method when phase I terminates.
7. Write down the dual of the following LPP :

   Maximize $z = x_1 + 2x_2 + x_3$

   subject to the constraints :

   $$2x_1 + x_2 - x_3 \le 2$$
   $$-2x_1 + x_2 - 5x_3 \ge -6$$
   $$4x_1 + x_2 + x_3 \le 6$$
   $$x_1, x_2, x_3 \ge 0.$$
8. Give the mathematical formulation of the transportation problem.
9. Define loop in a transportation table.
10. State the necessary condition for the existence of feasible solution to the transportation problem.
11. What is an assignment problem ?
12. State Konig theorem.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions. Each question carries* 3 *marks.*
All questions can be attended. Overall Ceiling 24.

13. A manufacturer produces two types of models M_1 and M_2. Each model of the type M_1 requires 4 hours of grinding and 2 hours of polishing whereas each model of the type M_2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works 40 hours a week and each polisher works for 60 hours a week. Profit on M_1 model is Rs. 3.00 and on model M_2 is Rs. 4.00. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week.

Turn over

14. Show that the set $S = \{(x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \le 4, x_1 + 2x_2 - x_3 \le 1\}$ is a convex set.

15. Rewrite in standard form the following linear programming problem :

    Minimize $Z = 12x_1 + 5x_2$

    subject to the constraints

    $$6x_1 + 3x_2 \ge 15$$
    $$7x_1 + 2x_2 \le 14$$
    $$x_1, x_2 \ge 0.$$

16. Verify the Minimax theorem for the function $f(x) = \{10, 8, 5, 2, 1\}$.

17. State the general rules for converting any primal LPP into its dual.

18. Verify that the dual of dual is primal for the following LPP :

    Maximize $z = 8x_1 + 3x_2$

    subject to the constraints :

    $$x_1 - 6x_2 \le 2$$
    $$5x_1 + 7x_2 = -4$$
    $$x_1, x_2 \ge 0.$$

19. Prove that in a balanced transportation problem having m origins and n destinations $(m, n \ge 2)$, the exact number of basic variables is $m + n - 1$.

20. Prove that a set X of column vectors of the co-efficient matrix of a transportation problem is linearly dependent if their corresponding cells in the transportation table contains a loop.

21. Write all the steps for the North-West corner rule of solving a transportation problem.

22. How to solve the degeneracy in transportation problem ?

23. Write steps for solving assignment problem by Hungarian method.

24. State the difference between transportation problem and assignment problem.

(8 × 3 = 24 marks)

Section C

Answer at least **five** *questions.*

Each question carries 6 *marks.*

All questions can be attended.

Overall Ceiling 30.

25. Prove that the set of all convex combinations of a finite number of vectors $x_1, x_2, \dots x_k$ in R^n is a convex set.

26. Let S be a convex subset of the plane, bounded by lines in the plane. Prove that a linear function $z = c_1x_1 + c_2x_2$, where c_1 and c_2 are scalars, attains its extreme values at the vertices of S only.

27. Solve the following LPP by graphical method :

Maximize $Z = 5x_1 + 7x_2$

subject to the constraints :

$$x_1 + x_2 \leq 4$$
$$3x_1 + 8x_2 \leq 24$$
$$10x_1 + 7x_2 \leq 35$$
$$x_1, x_2 \geq 0.$$

28. Find all the basic solution to the system of linear equations : $x_1 + 2x_2 + x_3 = 4$ and $2x_1 + x_2 + 5x_3 = 5$. Are the solutions degenerate ?

29. Use simplex method to solve the following LPP :

Maximize $z = 4x_1 + 10x_2$

subject to the constraints :

$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1, x_2 \geq 0.$$

30. Use Charne's penalty method to :

Minimize $z = 2x_1 + x_2$

subject to the constraints :

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$

31. Prove that if the k^{th} constraint of the primal problem is an equality then the k^{th} dual variable will be unrestricted in sign.

32. A company has 5 jobs to be done on five machines. Any job can be done on any machine. The cost of doing the jobs on different machines are given below. Assign the jobs for different machines so as to minimize the total cost :

	Machines				
Jobs	A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

Turn over

33. Determine an initial feasible solution to the following transportation problem using the Vogel's approximation method :

	A_1	B_1	C_1	D_1	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	950

(5 × 6 = 30 marks)

Section D

Answer any **one** *question.*

The question carries 14 *marks.*

34. Prove that there is a one-to-one correspondence between the optimum solutions to the General LPP and its reformulated LPP.

35. Solve the LPP by simplex method :

Maximize $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

subject to the constraints :

$$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$
$$7x_1 + x_4 \le 70$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

36. Find the optimal solution of the following transportation problem whose cost matrix is given as under :

	D_1	D_2	D_3	D_4	Supply
O_1	1	5	3	3	34
O_2	3	3	1	2	15
O_3	0	2	2	3	12
O_4	2	7	2	4	19
Required	21	25	17	17	80

(1 × 14 = 14 marks)