C 1247 (Pages : 4) Name..

Reg. No.....................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2021

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. An analytic function with constant modulus is ——————.
2. Fill in the blanks : The real part of $f(z) = \ln(z)$ is ——————.
3. Fill in the blanks : $f(z)$ is singular at infinity if ——————.
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-1)^2}{z^2(z^2+1)}$.
5. Define harmonic function.
6. Give an example of a complex function which is nowhere analytic.
7. Fill in the blanks : $\text{Res}_{z=0} \cot z =$ ——————.
8. State Morera's theorem.
9. Solution of $\sinh(z) = 0$ is ——————.
10. The radius of convergence of the series $\sum_{n=0}^{\infty} \dfrac{(z-i)^n}{n!}$ is ——————.
11. Fill in the blanks : For $f(z) = \dfrac{\tan z}{z}$; $z = 0$ is ——————.
12. Find the value of $\text{Log}(-10i)$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*

Each question carries 6 *marks.*

All questions can be attended.

Overall Ceiling 48.

13. Prove or disprove : $|\sin(z)| \leq 1$ for all complex numbers z. Justify your claim.

14. Verify Cauchy-Riemann equations for the function $f(z) = \ln z$.

15. If $f(z) = u + iv$ is analytic then derive the condition under which $v + iu$ is analytic .

16. Show that the poles of an analytic function are isolated.

17. Evaluate $\oint_{|z|=1} \bar{z}dz$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty} \frac{n!(z-i)^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^2$ when the contour of integration is the circle with centre at origin and radius 5 units.

20. Locate the singular points if any, of $f(z) = \frac{1}{\sin(\pi/z)}$ in the complex plane.

21. Find all the solutions of $e^z = -10$.

22. Evaluate the integral of $f(z)$ around the circle $|z| = 2$, where $f(z) = \frac{\cos z}{z^2}$.

23. Find the Residue of $\tan z$ at $z = \pi/2$.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Find the real and imaginary parts of the function $f(z) = \cos(z)$.

26. Find the principal value of $(1-i)^{1+i}$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Evaluate $\oint_C \frac{z^2+1}{\left(z^2-1\right)}$, where $C = |z-1| = 1$.

28. Show that $\tan^{-1}(z) = \frac{i}{2}\log\frac{i+z}{i-z}$.

29. Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor series about $z = 1$.

30. State and prove Liouvillie's theorem.

31. Find the harmonic conjugate of $u(x, y) = \text{Re}\left(f(z)\right) = \frac{x}{x^2+y^2}$.

32. Derive the polar form of Cauchy-Riemann Equations.

33. State and prove the Cauchy's Integral formula.

34. Find an analytic function in terms of z, whose real part is $e^x\left(x\cos y - y\sin y\right)$.

35. Find the residues of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its poles.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) State and prove Laurents theorem.

(b) Expand $f(z) = \frac{1}{(z+1)(z+2)}$ as a Laurent series valid for $0 < |z+1| < 2$.

Turn over

37. (a) State and prove Cauchy's Residue theorem.

(b) Evaluate $\oint_{|z|=1} \frac{\exp z}{\cos \pi z} dz$.

38. (a) Evaluate using the method of residues $\int_0^{2\pi} \frac{1}{a + b \cos \theta} d\theta$.

(b) Evaluate $\int_0^{\infty} \frac{x^2}{\left(x^2 + a^2\right)^2} dx, a > 0$.

(1 × 15 = 15 marks)