ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER B.Sc (FYUGP) DEGREE EXAMINATION, November 2024
B.Sc Mathematics
MAT1CJ101 : DIFFERENTIAL CALCULUS
2024 Admission Onwards
(Credits: 4)

Time : 2 Hours   **Maximum Marks : 70**

Section A
*[Answer **all**. Each question carries 3 Marks]*   **(Ceiling: 24 Marks)**

1. Find the domain and range of each of the following functions   [BTL1]
   i) $f(x) = 1 + x^2$
   ii) $g(x) = 1 - \sqrt{x}$

2. Find $\lim_{x \to 1} \dfrac{-1}{3x - 1}$ [BTL3]

3. If $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \to 0} f(x)$ [BTL4]

4. Define $h(2)$ in a way that extends $h(t) = \dfrac{t^2 + 3t - 10}{t - 2}$ to be continuous at $t = 2$ [BTL4]

5. Find the first and second derivatives of $w = 3z^7 - 7z^3 + 21z^2$ [BTL1]

6. Find the derivative of $g(t) = \tan(5 - \sin 2t)$ [BTL3]

7. State the first derivative theorem for local extreme values.   [BTL2]

8. If f is continuous on $[a, b]$, differentiable on (a, b) and $f' > 0$ at each point of   [BTL4]
   (a, b), then prove that f increases on $[a, b]$

9. Describe the concavity of $y = x^2$ [BTL1]

10. Find $\lim_{x \to -\infty} \dfrac{2x^2 - 3}{7x + 4}$ [BTL3]

Section B
*[Answer **all**. Each question carries 6 marks]*   **(Ceiling: 36 Marks)**

11. Graph the function $y = x^2$ over the interval $[-2, 2]$ [BTL2]

Turn Over

12. Suppose that $\lim_{x \to -2} p(x) = 4$, $\lim_{x \to -2} r(x) = 0$ and $\lim_{x \to -2} s(x) = -3$ [BTL4]
Find :-
i) $\lim_{x \to -2}(p(x) + r(x) + s(x))$

ii) $\lim_{x \to -2} p(x)r(x)s(x)$

iii) $\lim_{x \to -2} \dfrac{-4p(x) + 5r(x)}{s(x)}$

13. Differentiate the function $k(x) = \dfrac{1}{2 + x}$ and find the slope of the tangent   [BTL3]
line at $x = 2$

14. Find the tangent and normal to the curve $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$ [BTL5]

15. Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.   [BTL4]

16. Define an increasing function and a decreasing function. Find an interval in   [BTL1]
which $f(x) = x^2$ decreases.

17. State the second derivative test for local extreme values. Also give an example.   [BTL1]

18. Find the asymptotes of the graph of $f(x) = \dfrac{-8}{x^2 - 4}$ [BTL4]

Section C
*[Answer **any one**. Each question carries **10** Marks]*   **(1x10=10 Marks)**

19. Evaluate the following limits   [BTL3]
i) $\lim_{x \to -5} \dfrac{x^2 + 3x - 10}{x + 5}$

ii) $\lim_{h \to 0^+} \dfrac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$

20. i) State and prove Mean value theorem.   [BTL2]
ii) Verify the hypothesis of the Mean value theorem for the function $f(x) = x^2$
on the interval $[0, 2]$

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