D 110209 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2024

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. Prove that there does not exist a rational number r such that $r^2 = 2$.
2. Determine the set A of $x \in \mathbb{R}$ such that $|2x+3| \leq 7$.
3. If $a, b \in \mathbb{R}$, prove that $|a+b| \leq |a| + |b|$.
4. State the supremum property of $\mathbb{R}$.
5. If $S = \left\{\frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N}\right\}$, find inf S and sup S.
6. State and prove Archimedean property.
7. Let x and y be real numbers with $x < y$, prove there exists an irrational number z such that $x < z < y$.
8. State and prove squeeze theorem.
9. If a sequence (x_n) of real numbers converges to a real number x, prove that any subsequence (x_{nk}) of (x_n) also converges to x.

Turn over

10. Prove that every Cauchy sequence of real numbers is bounded.

11. Let (x_n) and (y_n) be two sequence of real numbers and suppose that $x_n \le y_n$ for all $n \in \mathbb{N}$. If $\lim x_n = +\infty$, prove that $\lim y_n = +\infty$.

12. Prove that the intersection of any finite collection of open sets in $\mathbb{R}$ is open.

13. Compute $\left(1+\sqrt{3}i\right)^9$.

14. Find the real and imaginary parts of $f(z) = z^2 - (2+i)z$ as a function of x and y.

15. Show that the complex function $f(z) = z + 3i$ is a one to one on the entire complex plane and find a formula for its inverse function.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *marks.*

16. State and prove Cantor's theorem.

17. Let a and b be positive real numbers, prove that $\sqrt{ab} \le \frac{a+b}{2}$ and the equality occurs if and only if $a = b$.

18. State and prove density theorem.

19. Prove that unit interval $[0,1]$ is not countable.

20. State and prove monotone convergence theorem.

21. Let $F \subseteq \mathbb{R}$; prove that the following are equivalent :

    (a) F is a closed subset of $\mathbb{R}$;

    (b) If $X = (x_n)$ is any convergent sequence of element in F, then lim X belongs to F.

22. Find an upper bound for $\left|\frac{-1}{z^4 - 5z + 1}\right|$ if $|z| = 2$.

23. For any *two* complex numbers, prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|\right)^2$.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. a) If A_m is a countable set for each $m \in \mathbb{N}$, prove that $A = \bigcup_{m=1}^{\infty} A_m$ is countable.

b) State and prove Bernoulli's inequality.

25. a) State and prove monotone convergence theorem.

b) Let $s_1 = 1$ and $s_{n+1} = \frac{1}{2}\left(s_n + \frac{a}{s_n}\right)$ for $n \in \mathbb{N}$. Prove that (s_n) converges to $\sqrt{a}$.

26. a) Prove that every contractive sequence is a Cauchy sequence.

b) Let $f_1 = 1, f_2 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Define $x_n = \frac{f_n}{f_{n+1}}$. Prove that $\lim x_n = \frac{-1+\sqrt{5}}{2}$.

27. Find a complex linear function that maps the equilateral triangle with vertices $1+i, 2+i$ and $\frac{3}{2} + \left(1 + \frac{1}{2}\sqrt{3}\right)i$ onto the equilateral triangle with the vertices $i, \sqrt{3} + 2i$ and $3i$.

(2 × 10 = 20 marks)