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D 110200 (Pages : 3) Name.......................................

Reg. No...................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2024

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2019 Admission only)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

All questions can be answered.
Each question in this section has 2 marks.
Ceiling is 25.

1. State Cantor's theorem.
2. Determine the set A of all real numbers x such that $2x+3\leq 6$.
3. Let $S=\left\{1-\dfrac{(-1)^n}{n}\;/n\in\mathbb{N}\right\}$. Fin inf S and sup S.
4. If $a, b\in\mathbb{R}$, prove that $|a+b|\leq|a|+|b|$.
5. State density theorem.
6. List five terms of the sequence $x_1=1, x_{n+1}=3x_n+1$.
7. Using the definition of the limit of a sequence, show that lim $\dfrac{3n+1}{2n+5}=\dfrac{3}{2}$.
8. Give an example of a monotone sequence which is not convergent.
9. Using divergence criteria, prove that $\left((-1)^n\right)$ is divergent.
10. Prove that a cauhy sequence of real numbers is bounded.

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11. Define contractive sequence.

12. Determine which of the complex numbers $z_1 = 10 + 8i, z_2 = 11 - 6i$ is closest to $1 + i$.

13. Express $-\sqrt{3} - i$ in polar form.

14. Find the real and imaginery parts of $f(z) = z^2 - (2 + i)z$ as functions of x and y.

15. Define continuity of a complex function $f(z)$ at z_0.

Section B

All questions can be answered.
Each question in this section has 5 *marks.*
Ceiling is 35.

16. State and prove Bernoulli's inequality.

17. Prove that an upper bound u of a nonempty set S in $\mathbb{R}$ is the supremum of S if and only if for every $\in > 0$ there exists an $S_\in \in S$ such that $u - \in < S_\in$.

18. State and prove Archimedean property of $\mathbb{R}$.

19. Prove that the set $\mathbb{R}$ of real numbers is not countable.

20. If $X = (x_n)$ is a sequence of real numbers, prove that there is a subsequence of X that is monotone.

21. State and prove Cauchy convergence criterion for sequences.

22. Find an upper bound for $\left| \frac{-1}{z^4 - 5z + 1} \right|$ if $|z| = 2$.

23. Prove that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$. Using this find the argument of $i(-\sqrt{3} - i)$.

Section C

Answer any **two** *questions.*
Each question in this section has 10 *marks.*

24. (a) Let a and b be positive real numbers. Prove that $\sqrt{ab} \leq \frac{a+b}{2}$ and the equality occures if and only if $a = b$.

    (b) Find all $x \in \mathbb{R}$ that satisfy the equation $|x+1|+|x-2|=7$.

25. Prove that there exits a positive real number x such that $x^2 = 2$.

26. (a) Prove that every contractive sequence is convergent.

    (b) Let $f_1 = f_2 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Define $x_n = \frac{f_n}{f_{n-1}}$. Prove that $\lim x_n = \frac{-1+\sqrt{5}}{2}$.

27. (a) Find the four fourth roots of $z = 1 + i$.

    (b) Solve the simultaneous equations $|z| = 2$ and $|z-2| = 2$.

(2 × 10 = 20 marks)