

D 110212

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Name.....

Reg. No.....

**FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION  
NOVEMBER 2024**

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours

Maximum : 60 Marks

**Section A**

*Not more than 20 marks can be earned from this unit.  
Each question carries 2 marks.*

1. State Reflection Property of the Parabola.
2. Determine the equation of the tangent to the ellipse with parametric equations  
 $x = 3\cos t, y = \sin t$   
at the point with parameter value  $t = \pi/4$ .
3. Prove that Euclidean-congruence is a symmetric relation.
4. Give the inverse of the affine transformation  $t(x) = Ax + b$ .
5. Find the quotient and remainder obtained when  $f(x) = 2x^7 - 3x^6 + x^5 - 3x^4 + 5x^3 - 4x^2 + 2x - 1$  is divided by  $g(x) = 2x^3 - 3x^2 + x - 1$ .
6. Calculate the values of the polynomial  $4x^3 - 7x^2 + 5x + 3$  and their derivatives for the value of  $x = -2$ .
7. State the Fundamental theorem of Algebra.
8. Verify that  $i$  is a zero of  $f(x) = x^3 + 2x - i$
9. How many real roots has the equation  $x^4 - 4ax + b = 0$  ?

Turn over

10. Verify that the equation  $x^3 - 7x + 7 = 0$  has roots in the interval  $\left(\frac{3}{2}, 2\right)$ .
11. State True/False : Let  $\alpha, \beta, \gamma$  are the roots of the equation  $f(x) = 0$ , then  $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \dots$  are the roots of the equation  $f\left(\frac{1}{x}\right) = 0$ .
12. State True/False : If the equation contains only even powers of  $x$  and the co-efficients are all of the same sign, the equation has no real root.

### Section B

*Not more than 30 marks can be earned from this unit.  
Each question carries 5 marks.*

13. Prove that  $2 \times 2$  matrix  $\mathbf{P}$  represents a rotation of  $\mathbb{R}^2$  about the origin if and only if it satisfies the following two conditions :
- $\mathbf{P}$  is orthogonal ;
  - $\det \mathbf{P} = 1$ .
14. Determine the affine transformation which maps the points  $(2, 3)$ ,  $(1, 6)$  and  $(3, -1)$  to the points  $(1, -2)$ ,  $(2, 1)$  and  $(-3, 5)$ , respectively.
15. Show that the roots of the equation

$$x^3 + px^2 + qx + r = 0$$

are in arithmetic progression if  $2p^3 - 9pq + 27r = 0$ .

16. If  $\alpha, \beta, \gamma$  are roots of  $x^3 + px^2 + qx + r = 0$ , find the values of  $\sum \frac{1}{\beta\gamma}$  in terms of co-efficients of the equation.
17. Find an upper limit of the positive roots of the equation
- $$x^5 - 7x^4 - 100x^3 - 1000x^2 + 10x - 50 = 0.$$

18. Find the rational roots of the equation  $6x^4 - 7x^3 + 8x^2 - 7x + 2 = 0$ .

19. Using Descartes' Rule of signs, show that the equation :

$$x^6 - x^3 + 2x^2 - 3x - 1 = 0$$

has four imaginary roots.

### Section C

*Answer any **one** question.*

*Each question carries 10 marks.*

20. (a) Prove that an affine transformation maps parallel straight lines to parallel straight lines.

(b) If  $\alpha, \beta$  and  $\gamma$  are the roots of the equation  $x^3 + ax^2 + bx + c = 0$ , form the equation whose roots are  $\alpha\beta, \beta\gamma$  and  $\gamma\alpha$ .

21. (a) Solve the biquadratic equation  $x^4 - 3x^2 + 6x - 2 = 0$ .

(b) Solve  $x^3 - 6x^2 + 3x - 2 = 0$  by Cardano's method.

(1 × 10 = 10 marks)