QP Code:P24A037

Reg. No : .....

Name : .....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20
I SEMESTER M.Sc(CBCSS -PG)DEGREE EXAMINATION, November 2024
M.Sc Mathematics
MTH1C04 : DISCRETE MATHEMATICS
2024 Admission Onwards

Time:3 Hours **Maximum Weightage:30**

Part A

*(Answer **all** questions. Weightage **1** for each question)*

1. Define a partial order relation on a set X with an example. [BTL2]
2. Define Complete Disjunctive Normal Forms. [BTL2]
3. Define a co-atom of a Boolean Algebra [BTL2]
4. Explain the diagramatic representation of a graph. [BTL4]
5. Explain edge connectivity. [BTL4]
6. Define the dual of a graph and give an example. [BTL1]
7. Explain concatenation of two strings. [BTL4]
8. Define the language accepted by an nfa. [BTL3]

(8x1 = 8 Weightage)

Part B

*(Answer **any two** questions from each module. Weightage **2** for each question)*

Unit-I

9. Prove that if a subset of a partially ordered set has a maximum element, then it is unique. [BTL1]
10. Let $(X, +, \cdot, ')$ be a Boolean Algebra, $x, y \in X$, define $x \leq y$ if $x \cdot y' = 0$. Prove that [BTL2]
    i) $\leq$ makes the underlying set of a Boolean Algebra into a lattice
    ii) 0 and 1 are the minimum and the maximum elements of this lattice
11. Prepare the table of values of the Boolean function [BTL3]
    $f(x_1, x_2, x_3) = x_1' x_2 (x_1' + x_2 + x_1 x_3)$

Unit-II

12. For a connected plane graph G, prove that $n - m + f = 2$, where f denotes the number of faces of G [BTL3]
13. A vertex v of a connected graph G with atleast three vertices is a cut vertex of G if and only if there exist vertices u and w of G, distinct from v, such that v is in every $u - w$ path in G [BTL3]
14. i) Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges [BTL3]
    ii) Prove that in any graph G, the number of vertices of odd degrees is even.

Turn Over

Unit-III

15. Find a grammar that generates $L = \{a^n b^{n+1} : n \geq 0\}$ [BTL1]

16. Let $M = (Q, \sum, \delta, q_0, F)$ be a deterministic finite accepter, and let G_M be its associated transition graph. Then prove that for every $q_i, q_j \in Q$ and $w \in \sum^+$, $\delta^*(q_i, w) = q_j$ if and only if there is in G_M a walk with label w from q_i to q_j [BTL1]

17. Let G_M be the transition graph for some dfa M, Prove the following [BTL2]
    i) If $L(M)$ is infinite, then G_M must have atleast one cycle for which there is a path from the initial vertex to some vertex in the cycle and a path from some vertex in the cycle to some final vertex.
    ii) If $L(M)$ is finite, then no such cycle exists.

(6x2 = 12 Weightage)

Part C

*(Answer **any two** questions. Weightage **5** for each question)*

18. Let (X,+,·,') be a Boolean Algebra. Then prove the following properties for all x,y,z of X [BTL2]
    i) x+x = x and x·x = x
    ii) x+1 = 1 and x·0 = 0
    iii) x+x·y = x and x·(x+y) = x
    iv) x+(y+z) = (x+y)+z and x·(y·z) = (x·y)·z
    v) If x+y = 1 and x·y = 0 then y = x'
    vi) (x')' = x

19. For a connected graph G, prove that the following statements are equivalent [BTL3]
    i) G is Eulerian
    ii) The degree of each vertex of G is an even positive integer
    iii) G is an edge-disjoint union of cycles

20. For any loopless connected graph G, prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$ [BTL2]

21. Let L be the language accepted by a nondeterministic finite accepter $M_N = (Q_N, \sum, \delta_N, q_0, F_N)$. Then prove that there exists a deterministic finite accepter $M_D = (Q_D, \sum, \delta_D, \{q_0\}, F_D)$ such that $L = L(M_D)$ [BTL2]

(2x5 = 10 Weightage)

< ********** >