QP Code : P24A019

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER M.Sc. (CBCSS-PG) DEGREE EXAMINATION, November 2024
MSc Mathematics
MTH1C02 : Linear Algebra
2024 Admission Onwards

Time : 3 Hours **Maximum Weightage : 30**

Part A

*(Answer **all** questions. Weightage **1** for each question)*

1. If A is a $m \times n$ matrix with entries in the field F, Then prove that $rowrank(A) = columnrank(A)$. [BTL2]
2. Define an ordered basis for a vector space and write the standard ordered basis for $\mathcal{R}^n$. [BTL1]
3. Find two linear operators T and U on $\mathcal{R}^2$ such that $TU = 0$ but $UT \neq 0$. [BTL3]
4. If $\mathcal{C}$ is a field of complex numbers, which vectors in $\mathcal{C}^3$ are linear combinations of $(1, 0, -1), (0, 1, 1)$ and $(1, 1, 1)$? [BTL3]
5. Define a linear functional. Give an example. [BTL1]
6. Define the following for a linear operator $T : V - V$: [BTL1]
   i). Characteristic Polynomial
   ii). Characteristic Value
   iii). Characteristic Vector
   iv). Characteristic Space
7. Let S be any set of vectors in an inner product space V. then show that $S^{\perp}$ is a subspace of V. [BTL2]
8. For $\alpha = (x_1, x_2); \beta = (y_1, y_2)$ in $\mathcal{R}^2$ prove that $(\frac{\alpha}{\beta}) = x_1y_1 - x_2y_1 - x_1y_2 + 4x_2y_2$ is an inner product in $\mathcal{R}^2$. [BTL3]

(8x1 = 8 Weightage)

Part B

*(Answer **any two** questions from each module. Weightage **2** for each question)*

Unit-I

9. Prove that a non empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar c in $\mathcal{F}$, the vector $c\alpha + \beta$ is again in W. [BTL2]

Turn Over

10. Is the vector $(3, -1, 0, -1)$ in the subspace of $\mathcal{R}^5$ spanned by the vectors $(2, -1, 3, 2), (-1, 1, 1, -3)$ and $(1, 1, 9, -5)$. [BTL2]

Turn Over

11. Let V be the vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. If $T(A) = AB - BA$ is T a linear transformation from V into V? Justify your answer. [BTL4]

Unit-II

12. If W_1 and W_2 are subspaces of a finite dimensional vector space, then $W_1 = W_2$ if and only if $W_1^0 = W_2^0$. [BTL2]

13. If $f$and g are linear functionals on a vector space V, then show that g is a scalar multiple of f if and only if the nullspace of g contains the nullspace of f. [BTL4]

14. Let T be a linear operator on an n dimensional vector space V. Then prove that the characteristic and the minimal polynomial for T have same roots except for multiplicities. [BTL3]

Unit-III

15. Find a projection E which projects $\mathcal{R}^2$ onto the sub spacespanned by $(1, -1)$ along the sub spacespanned by $(1, 2)$. [BTL3]

16. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto W, $W^\perp$is the nullspace of E and $V = W \oplus W^\perp$. [BTL2]

17. Find an inner product on $\mathcal{R}^2$ such that $(e_1/e_2) = 2$. [BTL4]

(6x2 = 12 Weightage)

Part C

*(Answer **any two** questions. Weightage **5** for each question)*

18. Let V be an n-dimensional vector space over the field F, and let W be an $m-$ dimensional vector space over F. Then prove that the space $L(V, W)$ is finite-dimensional and has dimension mn. [BTL3]

19. Let V be a finite dimensional vector space over the field Fand let T be a linear operator on V. Then prove that T is diagonalisable if and only if the minimal polynomial for T has the form $p = (x - c_1) \cdots (x - c_k)$ where $c_1, c_2, \cdots, c_k$ are distinct elements of F. [BTL4]

20. State and prove Cayley Hamilton Theorem. [BTL4]

21. Prove that every finite dimensional inner product space has an orthonormal basis. [BTL2]

(2x5 = 10 Weightage)

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