QP Code:P24A039

Reg. No : .....
Name : .....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER M.Sc (CBCSS-PG) DEGREE EXAMINATION, November 2024
M.Sc Mathematics
MTH1C05 : Number Theory
2024 Admission Onwards

Time:3 Hours **Maximum Weightage:30**

Part A
*(Answer **all** questions. Weightage **1** for each question)*

1. Prove that $(f * g)^{-1} = f^{-1} * g^{-1}$, if f(1)$\neq$ 0 and g(1)$\neq$0. [BTL3]
2. If the same primes divide m,n. Prove that $n\phi(m) = m\phi(n)$. [BTL2]
3. Prove that if f and g are completely multiplicative, then f/g is completely multiplicative. [BTL1]
4. Show that $\phi(n)$ is even for $n \geq 3$. Moreover if n has r distinct odd prime factors, then $2^r/\phi(n)$. [BTL3]
5. Define the "little oh" notation and prove that $f(x) = O(1) \implies f(x) = o(x)$. [BTL1]
6. Prove that $\lim_{x\to\infty} \frac{\pi(x)\log\pi(x)}{x} = 1 \implies \lim_{n\to\infty} \frac{P_n}{n\log n} = 1$. [BTL4]
7. Find the quadratic residue modulo 11. [BTL3]
8. Using digraph encipher transformation, encipher the word "ON". [BTL3]

(8x1 = 8 Weightage)

Part B
*(Answer **any two** questions from each module. Weightage **2** for each question)*

Unit-I

9. Prove that $\sigma_\alpha^{-1}(n) = \sum_{d/n} d^\alpha \mu(d)\mu(\frac{n}{d})$, where $n \geq 1$. [BTL1]
10. For $x \geq 1$, prove that $\sum_{n\leq x} \mu(n)[\frac{x}{n}] = 1$ and $\sum_{n\leq x} \Lambda(n)[\frac{x}{n}] = \log[x]!$. [BTL3]
11. Show that $f(n) = \sum_{d/n} g(d) \implies g(n) = \sum_{d/n} f(d)\mu(\frac{n}{d})$ and conversely. [BTL4]

Unit-II

12. If $x \geq 1$, show that $\lim_{x\to\infty}(\frac{M(x)}{x} - \frac{H(x)}{x\log x}) = 0$. [BTL2]
13. Prove that $\lim_{x\to\infty} \frac{\pi(x)\log\pi(x)}{x} = 1 \implies \lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$. [BTL4]

Turn Over

14. For all $x \geq 1$ prove that [BTL3]
    (i) $\sum_{n\leq x} \psi(\frac{x}{n}) = x\log x - x + O(\log x)$
    (ii) $\sum_{n\leq x} \vartheta(\frac{x}{n}) = x\log x + O(x)$.

Unit-III

15. Determine the odd primes for which 3 is a quadratic residue and for which it is a non-residue. [BTL3]
16. Prove that Legendre's symbol (n/p) is completely multiplicative function of n. [BTL2]

17. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix} (\mod 29)$. [BTL2]

(6x2 = 12 Weightage)

Part C

*(Answer **any two** questions. Weightage **5** for each question)*

18. Let f be multiplicative then prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n).f(n)$. [BTL2]

19. Prove that prime number theorem $\psi(x) \sim x \implies \lim_{x\to\infty} \frac{M(x)}{x} = 0$. [BTL4]

20. Solve the system of equations $x + 3y \equiv 1 \pmod{26}$
$7x + 9y \equiv 2 \pmod{26}$. [BTL3]

21. In the 27 - letter alphabet (with blank space = 26), use the affine enciphering transformation with n key a=13, b=9 to encipher the message " HELP ME ". [BTL3]

(2x5 = 10 Weightage)

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