QP Code : P24A029

Reg. No : ....

Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER M.Sc. (CBCSS-PG) DEGREE EXAMINATION, November 2024

MSc Mathematics

MTH1C03 : Real Analysis- I

2024 Admission Onwards

Time : 3 Hours **Maximum Weightage : 30**

Part A

*(Answer **all** questions. Weightage **1** for each question)*

1. Show that set of all rational numbers is countable. [BTL1]
2. Show that arbitrary union of open sets is open. [BTL2]
3. If f is a continuous real function on a metric space X, then prove that the zero set of f, Z(f) is closed. [BTL3]
4. Show that L'Hospitals' rule need not hold for Complex valued functions using a suitable example. [BTL3]
5. If f(x) = 0 for all irrational x and f(x) = 1 for all rational x, prove that f is not Riemann integrable on [a,b]. [BTL4]
6. Let f be defined on [a,b]. Show that, if f is differentiable at x in [a,b], then f is continuous at x. [BTL2]
7. Let γ be a curve in the complex plane, defined on [0, 2π] by $\gamma(t) = e^{2it}$. Prove that the length of γ is 4π. [BTL2]
8. Let f_n be a sequence of Riemann integrable functions such that $f_n \to f$. Is f Riemann integrable? Justify your answer. [BTL3]

(8x1 = 8 Weightage)

Part B

*(Answer **any two** questions from each module. Weightage **2** for each question)*

Unit-I

9. Show that compact subsets of metric spaces are closed. [BTL2]
10. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y. [BTL2]
11. Prove that in a metric space, a continuous function maps connected sets to connected sets. [BTL3]

Turn Over

Unit-II

12. If $f \in \mathscr{R}(\alpha)$ and $g \in \mathscr{R}(\alpha)$ on [a,b], then show that $|f| \in \mathscr{R}(\alpha)$ and $|\int_a^b f d\alpha| \leq \int_a^b |f| d\alpha$. [BTL2]

13. State and prove generalised mean value theorem for real functions. [BTL3]

14. Show that for a differentiable function f that has a local maximum at a point x in (a,b), $f'(x) = 0$. [BTL4]

Unit-III

15. Suppose $\{f_n\}$is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then show that f is continuous on E. [BTL3]

16. Prove that sequence $\{f_n\}$ converges to f with respect to the metric on $\mathscr{C}(X)$, (set of all complex valued continuous, bounded functions on X) if and only if $f_n \to f$ uniformly on K. [BTL3]

17. Show by a suitable example that every convergent sequence neednot contain a uniformly convergent subsequence. [BTL4]

(6x2 = 12 Weightage)

Part C

*(Answer **any two** questions. Weightage **5** for each question)*

18. (a) Show that Cantor set is perfect. [BTL2]
(b) Show that every k-cell is compact.

19. (i) Suppose α increases on [a,b], $a \leq x_0 \leq b$, α is continuous at x_0, $f(x_0) = 1$ and $f(x) = 0$ if $x \neq x_0$. Prove that $f \in \mathscr{R}(\alpha)$ and $\int f d\alpha = 0$. [BTL3]
(ii) Suppose $f \geq 0$, f is continuous on [a,b] and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0$ for all x in [a,b].

20. (a) Prove that continuity is a sufficient condition for Riemann Stieltjes integrability. [BTL3]
(b) Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on [a,b]. Let f be bounded real function on [a,b]. Then prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$.

21. (a)Suppose $\{f_n\}$ is a sequence of functions, differentiable on [a,b] and such that $\{f_n(x_0)\}$ converges for some point x_0 on [a,b]. If $\{f_n'\}$ converges uniformly on [a,b], prove that $\{f_n\}$converges uniformly on [a,b] to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x)$, a < x < b. [BTL3]
(b) Show that there exists a real continuous function on the real line which is nowhere differentiable.

(2x5 = 10 Weightage)

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