D 111972 (Pages : 3) Name..

Reg. No...............................

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE—2

(2019—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

All *questions can be attended.*
Each question carries 2 *marks.*

1. Differentiate the function $f(x) = \log\left(\dfrac{x}{\ln(x)}\right)$.

2. Find the derivative of $y = \log\left(\left|\sec(x) + \tan(x)\right|\right)$.

3. Find $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{1 - \cos(2x)}$.

4. Show that $\cosh^2 x - \sinh^2 x = 1$.

5. Find $\lim\limits_{n \to \infty} e^{\frac{1+n}{1-n}}$.

6. Determine whether the sequence $a_n = 1 + (-1)^n / n^2$ converges or diverges. If it converges, find the limit.

7. Express $.111\bar{1}...$ as a rational number.

8. State the Squeeze Theorem.

9. By using the power series expansion of $\sin x$, show that $\dfrac{d}{dx}(\sin x) = \cos x$.

10. Find the Maclaurian series expansion of $\dfrac{1}{1 + x}$.

11. Find the rectangular equation of a curve whose parametric equation is $x = t + 1$, $y = t^2 - 1$.

12. Find the equation of the tangent to the ellipse $x = 3\cos t$, $y = 2\sin t$ at $t = \pi/4$.

13. Find an equation of the line that passes through the point $(-1, 0, 2)$ and is parallel to the vector $\langle 1, 5, -4 \rangle$.

Turn over

14. Find the equation of the surface $z = x^2 + y^2$ in cylindrical co-ordinates.

15. Find $r'(t)$ if $r(t) = 2\cos ti + 3\sin tj + 3tk$.

<div align="right">

$(15 \times 2 = 30 \text{ marks})$
Max. Ceiling : 25 marks
</div>

Section B

All questions can be attended.
Each question carries 5 marks.

16. Evaluate :

   (i) $\lim\limits_{x \to 0} \dfrac{\tan 5x}{\sin 2x}$.

   (ii) $\lim\limits_{x \to 0} \dfrac{x^3 - 3\sin^2 x}{x^2}$.

17. Find the derivative of $y = (\cos x)^x$.

18. Find $\int x^2 e^{-x} dx$.

19. Use the integral test to determine the series $\sum_1^\infty \dfrac{1}{n^2}$ converge or diverge.

20. Find the interval of convergence and radius of convergence of the power series $\sum_0^\infty \dfrac{x^{2n+1}}{(2n+1)!}$.

21. Find the Maclaurian series for $\dfrac{1}{\sqrt{1-x}}$ and determine its interval of convergence.

22. Find $\dfrac{d^2 y}{dx^2}$ for the parametric equation $x = a\cos t, y = b\sin t$.

23. Identify and sketch the graph of the surface $x^2 - x^2 - y^2 = 1$.

<div align="right">

$(8 \times 5 = 40 \text{ marks})$
Max. Ceiling : 35 marks
</div>

Section C

*Answer any **two** questions.*
Each question carries 10 marks.

24. (i) Show that $\int \dfrac{dx}{\sqrt{4x^2 - 9}}\, dx = \dfrac{1}{2}\cosh^{-1}\left(\dfrac{2x}{3}\right), x > 3/2$.

   (ii) Find $\int\limits_{-\infty}^{0} \dfrac{e^x}{\sqrt{1 + e^{2x}}}\, dx$.

   (iii) Find $\lim\limits_{x \to 0} (\tan x)^x$.

25. (i) Let C be the ellipse $r(t) = 3\cos t + 2\sin t$. Find T(t) and N(t) at $t = \pi/4$.

    (ii) Find the curvature of the curve $r(t) = ti + \dfrac{1}{t}j$ at $t = 1$.

26. (i) Find the total arc length of the cardioid $r = 1 - \cos\theta$.

    (ii) Find the area of the cardioid $r = 1 + \cos\theta$.

27. A shell fired from a cannon, has a muzzle speed of 80 ft/s. The barrel amkes an angle of 45° with the horizontal and, the barrel opening is assumed to be at ground level.

    (a) Find parametric equation for the shell's trajectory.

    (b) How high does the shell rise ?

    (c) How far does the shell travel horizontally ?

    (d) What is the speed of the shell at its point of impact with the ground.
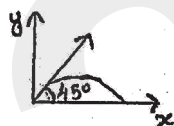
(2 × 10 = 20 marks)

D 111972-A   (Pages : 5)   Name.......................................

Reg. No..................................

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE—2

(2019—2023 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.

2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.

3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.

4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 3B 03—CALCULUS OF SINGLE VARIABLE—2

(Multiple Choice Questions for SDE Candidates)

1. $\dfrac{d}{dn}(\ln kx) =$ ————.

    (A) kx.

    (B) $\dfrac{1}{kx}$.

    (C) $\dfrac{1}{x}$.

    (D) $\dfrac{k}{\ln x}$.

2. Range of $\cos hx$ is ————.

    (A) $[-1, 1]$.

    (B) $(-\infty, \infty)$.

    (C) $(-\infty, 1]$.

    (D) $(-\infty, 1)$.

3. $\dfrac{d}{dx}\cosh^{-1}\left(x^2\right) =$ ————.

    (A) $2x\sinh^{-1}\left(x^2\right)$.

    (B) $\dfrac{2x}{\sqrt{x^2-1}}$.

    (C) $\dfrac{x}{x^4-1}$.

    (D) $\dfrac{2x}{\sqrt{x^4-1}}$.

4. $\displaystyle \lim_{x\to 0}\dfrac{a^x - b^x}{x} =$ ————.

    (A) $\ln\left(\dfrac{a}{b}\right)$.

    (B) $\ln\left(\dfrac{b}{a}\right)$.

    (C) $\ln(ab)$.

    (D) ∞.

5. $\tan hx$ is ———— function.

    (A) Odd.

    (B) Even.

    (C) Neither even nor odd.

    (D) None of these.

6. The sequence $\left\{(-1)^{n+1}\right\}$ is :

    (A) Converges.

    (B) Diverges.

    (C) Has a convergent subsequence.

    (D) None.

7. The sequence $\left\{\dfrac{1}{n}\right\}$ is :

    (A) Diverges.   (B) Increasing.

    (C) Decreasing.   (D) None of these.

8. What is true ?

    (I) Every bounded sequence is converged.

    (II) Every converges sequence is bounded.

    (A) I and II are True.   (B) I is True II is False.

    (C) I is False II is True.   (D) Both are False.

9. $\underset{x\to\infty}{\text{Lim}}\ \dfrac{3^{n}}{n^{2}} =$ ————.

    (A) 3.   (B) $\dfrac{3}{2}$.

    (C) 1.   (D) ∞.

10. $\underset{n\to\infty}{\text{Lim}}\left(1+\dfrac{1}{n}\right)^{n} =$ ————.

    (A) 1.   (B) 0.

    (C) e.   (D) $\dfrac{1}{e}$.

11. The series $\sum\limits_{n=1}^{\infty} a_{n}$ converges, then :

    (A) $\underset{n\to\infty}{\text{Lim}}\ a_{n} = c$, a constant.   (B) $\underset{n\to\infty}{\text{Lim}}\ a_{n} = 0$.

    (C) $\underset{n\to\infty}{\text{Lim}}\ a_{n} = \infty$.   (D) $\underset{n\to\infty}{\text{Lim}}\ a_{n}$ does not exist.

12. Let $\{a_{n}\}$ of $\{b_{n}\}$ is such that $a_{n} \le b_{n}$. Then :

    (A) Σa_{n} converges if Σb_{n} converges.

    (B) Σb_{n} converges if Σa_{n} converges.

    (C) Σa_{n} converges if Σb_{n} diverges.

    (D) Σb_{n} diverges if Σa_{n} diverges.

Turn over

13. $\sum\limits_{n=1}^{\infty} \dfrac{n}{n^3+1}$ is :

    (A) Converges to 1.

    (B) Converges to 0.

    (C) Diverges.

    (D) None of these.

14. Let $\{a_n\}$ be a sequence of positive terms such that $a_n \geq a_{n+1}$ and $\lim\limits_{n\to\infty} a_n = 0$, then :

    (A) Σa_n converges.

    (B) $\sum(-1)^n a_n$ converges.

    (C) Σa_n converges but $\sum(-1)^n a_n$ diverges.

    (D) None of these.

15. The series $\sum\limits_{n=0}^{\infty} x^n$ is :

    (A) Converges absolutely for $|x| < 1$.

    (B) Converges for $|x| > 1$.

    (C) Has radius of converges ½.

    (D) None of these.

16. Which of the following represent parametric equation of a circle :

    (A) $(x = a\cos t, y = b\sin t)$.

    (B) $(x = a\cos t, y = a\sin t)$.

    (C) $(x = a\sec t, y = b\tan t)$.

    (D) None.

17. Tangent to the curve, $x = 4\sin t, y = 2\cos t$ at $t = \dfrac{\pi}{4}$ is :

    (A) $-\dfrac{1}{2}$.

    (B) $\dfrac{1}{2}$.

    (C) -1.

    (D) 1.

18. The polar co-ordinate equal to $\left(3, \dfrac{\pi}{4}\right)$ is :

    (A) $\left(-3, \dfrac{\pi}{4}\right)$.

    (B) $\left(-3, 9\dfrac{\pi}{4}\right)$.

    (C) $\left(3, 9\dfrac{\pi}{4}\right)$.

    (D) $\left(3, 5\dfrac{\pi}{4}\right)$.

19. The equation $r^2 = \sin 2\theta$ is symmetric about :

   (A) x - axis.   (B) y-axis.

   (C) Origin.   (D) The line $\theta = \dfrac{\pi}{4}$.

20. $\text{Log}_a \left(a^x \right) = $ ———.

   (A) a^x.   (B) a.

   (C) x^a.   (D) x.