

**D 111973****(Pages : 2)****Name.....****Reg. No.....****THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024****Mathematics****MTS 3C 03—MATHEMATICS—3****(2019—2023 Admissions)****Time : Two Hours****Maximum Marks : 60****Part A***All questions can be answered.**Each question carries 2 marks.**(Ceiling 20 marks)*

1. If  $r(t) = \langle f(t), g(t), h(t) \rangle$ , where  $f$ ,  $g$  and  $h$  are differentiable, then prove that  $r'(t) = \langle f'(t), g'(t), h'(t) \rangle$ .
2. The position of a moving particle is given by  $r(t) = t^2i + tj + \frac{5}{2}tk$ . Find  $v(2)$  and  $a(2)$ .
3. Describe the level surfaces of the function  $f(x, y) = y^2 - x$ .
4. If  $F = (x^2y^3 - z^4)i + 4x^5y^2zj + y^4z^6k$ , find  $\text{curl } F$ .
5. Evaluate  $\int_C xy^2 ds$  on the quarter-circle  $C$  defined by  $x = 4 \cos t$ ,  $y = 4 \sin t$ ,  $0 \leq t \leq \frac{\pi}{2}$ .
6. Find  $\int_C y dx + x dy$  on the curve  $y = x^3$  between  $(0, 0)$  and between  $(1, 1)$ .
7. Convert  $\left(8, \frac{\pi}{3}, 7\right)$  in cylindrical co-ordinates to rectangular co-ordinates.
8. Find the values of  $\ln(-2)$ .
9. Prove that  $\cosh^2 z + \sinh^2 z = 1$ .
10. Evaluate  $\int_C \bar{z} dz$ , where  $C$  is given by  $x = 3t$ ,  $y = t^2$ ,  $-1 \leq t \leq 4$ .
11. Evaluate  $\oint_C e^z dz$ , where  $C$  is the circle  $|z| = 2$ .
12. Derive Cauchy's inequality.

**Turn over**

**Part B**

*All questions can be answered.  
Each question carries 5 marks.  
(Ceiling 30 marks)*

13. Find the directional derivative  $f(x, y) = 2x^2y^3 - 6xy$  at  $(1, 1)$  in the direction of a unit vector whose angle with the positive  $x$ -axis is  $\frac{\pi}{6}$ .
14. Find an equation of the tangent plane to the graph of  $x^2 - 4y^2 + z^2 = 16$  at  $(2, 1, 4)$ .
15. Evaluate the double integral  $\iint_R e^{x+3y} dA$  over the region bounded by the graphs of  $y = 1$ ,  $y = 2$ ,  $y = x$  and  $y = -x + 5$ .
16. Evaluate  $\oint_C (x^5 + 3y)dx + (2x - e^{y^3})dy$ , where  $C$  is the circle  $(x - 1)^2 + (y - 5)^2 = 4$ .
17. Find the volume of the solid in the first Octant bounded by the graphs of  $z = 1 - y^2$ ,  $y = 2x$  and  $x = 3$ .
18. Solve the equation  $\cos z = 10$ .
19. Find an upper bound for the absolute value of  $\oint_C \frac{e^z}{z+1} dz$ , where  $C$  is the circle  $|z| = 4$ .

**Section C**

*Answer any **one** questions.  
The question carries 10 marks.*

20. Let  $S$  be the part of the cylinder  $z = 1 - x^2$  for  $0 \leq x \leq 1$ ,  $-2 \leq y \leq 2$ . Verify Stoke theorem for the vector field  $\mathbf{F} = xy \mathbf{i} + yz \mathbf{j} + xz \mathbf{k}$ . Assume  $S$  is oriented upward.
21. Find the moment of inertia about the  $z$ -axis of the homogeneous solid bounded between the spheres  $x^2 + y^2 + z^2 = a^2$  and  $x^2 + y^2 + z^2 = b^2$ ,  $a < b$ .

(1 × 10 = 10 marks)