QP Code: P25B008 **Reg. No : ....**
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

II SEMESTER (CBCSS - PG) DEGREE EXAMINATION, March 2025

M Sc Mathematics

MTH2C06 : ALGEBRA II

2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage 1 for each question.* ***(8x1 = 8 Weightage)***

1. If R is a ring with unity, and N is an ideal of R containing a unit, then show that N = R. [BTL1]
2. Is π^2 algebraic over $\mathbb{Q}(\pi)$? Why? [BTL3]
3. Find $irr(\sqrt{3-\sqrt{6}}, \mathbb{Q})$ and degree of the polynomial. [BTL3]
4. Find conjugates of $3+\sqrt{2}$ over $\mathbb{Q}$. [BTL2]
5. Let E be a finite extension of degree n over a finite field F.If F has q elements, then prove that E has q^n elements. [BTL1]
6. Find the number of primitive 18^{th} roots of unity in $GF(19)$. [BTL3]
7. Is regular 60-gon constructible? Justify [BTL3]
8. Find $\phi(1000)$. [BTL3]

Part B

*Answer **any two** questions from each module. Weightage 2 for each question.*
(6x2 = 12 Weightage)

Unit-I

9. Let E be an extension field of F,and let $\alpha \in E$, where α is algebraic over F. Then show that there is an irreducible polynomial $p(x) \in F[x]$ such that
$p(\alpha) = 0$
and if $f(\alpha) = 0$ for $f(x) \in F[x]$, with $f(x) \neq 0$, then $p(x)$ divides $f(x)$. [BTL2]
10. Find degree and basis of $\mathbb{Q}(2^{\frac{1}{3}}, \sqrt{3})$ over $\mathbb{Q}$ [BTL5]
11. Show that the field F of constructible real numbers consists precisely of all real numbers that we can obtain from $\mathbb{Q}$ by taking square roots of positive numbers a finite number of times and applying a finite number of field operations. [BTL2]

Turn Over

Unit-II

12. If F is a field of prime characteristic p, then prove that $(\alpha+\beta)^{p^n}=\alpha^{p^n}+\beta^{p^n}$ for all $\alpha,\beta\in F$ and all positive integers n. [BTL1]

13. Let F be a finite field of characteristic p.Prove that the map $\sigma_p : F \to F$ defined by $\sigma_p(a)=a^p$ for $a\in F$ is an automorphism of F and $F_{\{\sigma_p\}} \cong \mathbb{Z}_p$. [BTL2]

14. If E is a finite extension of F, then prove that E is separable over F if and only if each $\alpha\in E$ is separable over F. [BTL2]

Unit-III

15. Let K be a finite normal extension of a field F, with Galois group $G(K/F)$. For a field E, where $F\le E\le K$, let $\lambda(E)$ be the subgroup of $G(K/F)$ leaving E fixed. Prove that $[K:E]=|\lambda(E)|$ and $[E:F]=(G(K/F):\lambda(E))$, the number of left cosets of $\lambda(E)$ in $G(K/F)$. [BTL2]

16. Prove that the Galois group of the n^{th} cyclotomic extension of $\mathbb{Q}$ has $\phi(n)$ elements . [BTL3]

17. Show that the polynomial x^5-1 is solvable by radicals over $\mathbb{Q}$. [BTL3]

Part C

Answer ***any two*** *questions. Weightage* ***5*** *for each question.* ***(2x5 = 10 Weightage)***

18. State and prove Kronecker's Theorem. [BTL1]

19. State and prove conjugation isomorphism theorem. [BTL1]

20. Show that every finite field is perfect. [BTL3]

21. Let K be a finite normal extension of a field F, with Galois group $G(K/F)$. For a field E, where $F\le E\le K$, let $\lambda(E)$ be the subgroup of $G(K/F)$ leaving E fixed. [BTL2]
(a) Prove that E is a normal extension of F if and only if $\lambda(E)$ is a normal subgroup of G(K/F).
(b) Prove that when $\lambda(E)$ is a normal subgroup of $G(K/F)$, then $G(E/F)\cong G(K/F)/G(K/E)$.

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