QP Code: P25B017

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

II SEMESTER (CBCSS - PG) DEGREE EXAMINATION, MARCH 2025

M Sc Mathematics

MTH2C07 : REAL ANALYSIS II

2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each question.* ***(8x1 = 8 Weightage)***

1. Find two sets A and B such that $A \cap B = \phi$ but $\bar{A} \cap \bar{B} \neq \phi$. [BTL2]
2. Define outer measure of a set A of real numbers. Show that outer measure is translation invariant. [BTL2]
3. Define Borel set and G_δ set. Is G_δ set , a Borel set? Justify your answer. [BTL3]
4. Show that the measure of Cantor set is 0. [BTL3]
5. Define Riemann integrability. Give an example of a function that is not Riemann integrable. [BTL2]
6. If f is a non negative measurable function on E and if A and B are disjoint measurable subsets of E, prove that $\int_{A \cup B} f = \int_A f + \int_B f$ [BTL3]
7. With an appropriate example, show that Fatou's lemma holds as a strict inequality. [BTL2]
8. Show that $f(x) = \sqrt{x}$ is not Lipschitz but is absolutely continuous on [0,1]. [BTL3]

Part B

*Answer **any two** questions from each module. Weightage **2** for each question.*
(6x2 = 12 Weightage)

Unit-I

9. Show that every nonempty open set in **R** is the disjoint union of countable collection of open intervals. [BTL3]
10. Prove that there exists nonmeasurable sets. [BTL4]
11. If $\{f_n\}$ is a sequence of measurable functions with common domain E, show that inf $\{f_n\}$ is measurable. [BTL3]

Turn Over

Unit-II

12. Show that monotone convergence theorem is not true for decreasing sequence of nonnegative measurable functions. [BTL1]

13. Prove that $m\{x \in E | f(x) \geq \lambda\} \leq \frac{1}{\lambda} \int_E f$ for any $\lambda > 0$ and non negative measurable function f on E. [BTL3]

14. Let f be bounded function on a set E of finite measure. Show that f is Lebesgue integrable over E if and only if it is measurable. [BTL3]

Unit-III

15. Prove that a function of bounded variation on [0,1] is the difference of two increasing functions. [BTL1]

16. If f is increasing on closed bounded interval [a,b], then prove that for each $\alpha > 0$, $m^*\{x \in (a,b) | \bar{D}f(x) = \infty\} = 0$. [BTL3]

17. Show that an increasing function f is absolutely continuous on [a,b] if and only if $\int_a^b f' = f(b) - f(a)$. [BTL2]

Part C

Answer ***any two*** *questions. Weightage* ***5*** *for each question.* ***(2x5 = 10 Weightage)***

18. Show that outer measure of an interval is its length. [BTL2]

19. Define measurability of functions. Prove that sum and product of two measurable functions is measurable. [BTL3]

20. (i) Show that if f and g are integrable over E, then $\alpha f + g$ is integrable over E and $\int_E \alpha f + g = \alpha \int_E f + \int_E g$ [BTL2]
    (ii) If $f \leq g$ on E, then show that $\int_E f \leq \int_E g$.

21. Show that an increasing function on (a,b) is differentiable a.e on (a,b). [BTL3]

****