QP Code: P25B033

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

II SEMESTER (CBCSS-PG) DEGREE EXAMINATION, MARCH 2025
M Sc Mathematics
MTH2C09 : ODE AND CALCULUS OF VARIATIONS
2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each question.* ***(8x1 = 8 Weightage)***

1. Give confluent hypergeometric equation. [BTL1]
2. Show that $sinx = x[\lim_{a\to\infty} F(a, a, \frac{3}{2}, \frac{-x^2}{4a^2})]$. [BTL3]
3. Locate and classify the singular points of the differential equation $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$. [BTL2]
4. Explain the types of critical points of non linear system. [BTL1]
5. Prove that $\Gamma(p+1) = p!$. [BTL2]
6. Prove that $\frac{d}{dx} J_0(x) = -J_1(x)$. [BTL1]
7. Define Lipschitz condition and also state Peano's theorem. [BTL1]
8. What is an admissible function? [BTL1]

Part B

*Answer **any two** questions from each module. Weightage **2** for each question.*
(6x2 = 12 Weightage)

Unit-I

9. Express $sin^{-1}x$ in the form of a power series by solving the equation $y' = (1-x^2)^{-1/2}, y(0) = 0$ in two ways. [BTL1]
10. Transform the Chebyshev's equation $(1-x^2)y'' - xy' + p^2y = 0$, where p is a non- negative constant into a hypergeometric equation and find the general solution near $x = 1$. [BTL3]
11. Determine the nature of the point $x = \infty$ for Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ [BTL4]

Turn Over

Unit-II

12. Show that (0,0) is an asymptotically stable critical point for the system [BTL3]
$\frac{dx}{dt} = -3x^3 - y, \frac{dy}{dt} = x^5 - 2y^3$.

13. If the two solutions $x = x_1(t), y = y_1(t)$ and $x = x_2(t), y = y_2(t)$ of the homogeneous system $\frac{dx}{dt} = a_1(t)x + b_1(t)y, \frac{dy}{dt} = a_2(t)x + b_2(t)y$ are linearly independent on $[a, b]$ and [BTL4]
if $x = x_p(t), y = y_p(t)$ is any particular solution of non-homogeneous system $\frac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t), \frac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t)$ on this interval then prove that
$x = c_1x_1(t) + c_2x_2(t) + x_p(t), y = c_1y_1(t) + c_2y_2(t) + y_p(t)$
is the general solution of the non-homogeneous system on $[a, b]$.

14. Prove that $J_p(x) = \frac{x}{2p}[J_{p-1}(x) + J_{p+1}(x)]$. Then find $J_{-3/2}(x)$ and $J_{-5/2}(x)$. [BTL3]

Unit-III

15. Describe Picard's method of successive approximation for solving the initial value problem $y' = f(x, y), y(x_0) = y_0$. [BTL3]

16. State and prove Sturm seperation theorem. [BTL4]

17. Let $q(x)$ be a positive continuous function that satisfies [BTL2]
$0 < m^2 < q(x) < M^2$ on $[a, b]$.
If $y(x)$ is a nontrivial solution of $y'' + q(x)y = 0$ on $[a, b]$ and if x_1 and x_2 are successive zeros of $y(x)$ then prove that $\frac{\pi}{M} < x_2 - x_1 < \frac{\pi}{m}$. Further more if $y(x)$ vanishes at a and b and at n-1 points in (a,b) then prove that
$\frac{m(b-a)}{\pi} < n < \frac{M(b-a)}{\pi}$.

Part C

*Answer **any two** questions. Weightage **5** for each question.* ***(2x5 = 10 Weightage)***

18. a) Find the indical equation and its roots of the differential equation [BTL2]
$4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.
b) Find two independent Frobenius series solution of the equation
$xy'' - y' + 4x^3y = 0$.

19. a) Derive Rodrigue's formula for Legendre polynomials; [BTL4]
$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$.
b) Show that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{Bmatrix} 0 & \text{if} & m \neq n \\ \frac{2}{2n+1} & \text{if} & m = n \end{Bmatrix}$.

20. a) Find the general solution of the system, $\frac{dx}{dt} = 3x - 4y, \frac{dy}{dt} = x - y$. [BTL4]
b) Find the critical points, differential equation of the paths of the system and solve this equation to find the paths, $\frac{dx}{dt} = y(x^2 + 1), \frac{dy}{dt} = 2xy^2$.

21. Prove Picard's theorem. [BTL5]

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