QP Code: P25B036

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

II SEMESTER (CBCSS-PG) DEGREE EXAMINATION, MARCH 2025

M Sc Mathematics

MTH2C10 : OPERATIONS RESEARCH

2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each question.* ***(8x1 = 8 Weightage)***

1. Let $f(X) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$. Write $f(X)$ in quadratic form. [BTL2]

2. Write the following linear programming problem in standard form [BTL3]

$$\begin{aligned} \text{Minimize} \quad & f = x_1 + x_2 - x_3 \\ \text{Subject to} \quad & x_1 + x_2 \geq 2 \\ & x_1 - x_3 \leq 4 \\ & 2x_1 - x_2 + x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

3. Prove that the set S_F of feasible solutions if not empty, is a closed convex set bounded from below and has atleast one vertex. [BTL5]

4. Describe a transportation problem. [BTL1]

5. Describe a connected graph and a strongly connected graph. [BTL1]

6. What do we do in sensitivity analysis in linear programming problem? [BTL4]

7. Explain an integer vector, then describe an integer linear programming problem. [BTL1]

8. Examine the given pay off matrix for saddle point [BTL5]

$$\begin{pmatrix} 4 & -2 & -4 & -1 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & -2 & -2 \\ -1 & -3 & -3 & 1 \\ -3 & 2 & -2 & -3 \end{pmatrix}$$

Part B

*Answer **any two** questions from each module. Weightage **2** for each question.*
(6x2 = 12 Weightage)

Unit-I

9. Write an algorithm that constitutes one iteration leading from one basic feasible solution to another in simplex method. [BTL3]

Turn Over

10. Using graphical method, solve the following linear programming problem [BTL5]

Maximize $f(X) = 3x_1 + 5x_2$

Subject to $x_1 + 2x_2 \leq 20$

$x_1 + x_2 \leq 15$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

11. Prove that a vertex of S_F is a basic feasible solution. [BTL1]

Unit-II

12. i) Define the dual of a linear programming problem. [BTL3]

ii) Write the dual of the linear programming problem.

Maximize $f(X) = 2x_1 + x_2 - x_3$

Subject to $2x_1 - 5x_2 + 3x_3 \leq 4$

$3x_1 + 6x_2 - x_3 \geq 2$

$x_1 + x_2 + x_3 = 4$

$x_1, x_3 \geq 0, x_2$ unrestricted

13. Prove the following statement: [BTL4]

If the primal problem is feasible, then it has an unbounded optimum, if and only if the dual has no feasible solution and vice versa.

14. Describe a Caterer problem with an example. [BTL1]

Unit-III

15. Characterize the optimal solution of the integer linear programming problem [BTL5]

Minimize $f(X) = CX$

Subject to $X \in T_F$

16. i) Define a zero sum game [BTL3]

ii) Let $f(X,Y)$ be such that both $\max_X \min_Y f(X,Y)$ and $\min_Y \max_X f(X,Y)$ exist. Then prove that

$\max_X \min_Y f(X,Y) \leq \min_Y \max_X f(X,Y)$

17. Write an algorithm to find a minimum spanning tree. [BTL2]

Part C

*Answer **any two** questions. Weightage **5** for each question.* ***(2x5 = 10 Weightage)***

18. Solve the following linear programming problem [BTL4]

Minimize $f(X) = 4x_1 + 5x_2$

Subject to $2x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 5$

$x_1 + x_2 \geq 1$

$x_1 + 4x_2 \geq 2$

$x_1, x_2 \geq 0$

19. Prove that the necessary and sufficient condition for a set of column vectors $P_{i,j}$ in the matric $\bar{T}$ to be linearly independent is that the corresponding variables x_{ij} in the transportation array occupy cells a subset of which constitutes a loop. [BTL2]

20. Solve the transportation problem for minimum cost with the cost coefficients, demands and supplies as given in the following table [BTL5]

	D_1	D_2	D_3	D_4	a_i
O_1	3	2	5	4	25
O_2	4	1	7	6	35
O_3	7	8	3	5	30
b_j	10	18	20	42	

21. Let $f(X,Y)$ be such that both $\max_X \min_Y f(X,Y)$ and $\min_Y \max_X f(X,Y)$ exist. [BTL3]
Then show that the necessary and sufficient condition for the existence of a saddle point (X_0, Y_0) of $f(X,Y)$ is that
$f(X_0, Y_0) = \max_X \min_Y f(X,Y) = \min_Y \max_X f(X,Y)$

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