QP Code: P25B026

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

II SEMESTER CBCSS-PG) DEGREE EXAMINATION, MARCH 2025

M Sc Mathematics

MTH2C08 : TOPOLOGY

2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each question.* ***(8x1 = 8 Weightage)***

1. Consider $\mathbb{R}$ as a metric space. Give an example of a metric on $\mathbb{R}$ other than usual metric. [BTL3]
2. What is Sierpinski space? Is this space obtainable from a metric or even a pseudo metric? Justify your answer. [BTL1]
3. Prove that the class of all open intervals with rational end points form a base for usual topology on $\mathbb{R}$. [BTL3]
4. Consider the Discrete topology on the set $X = \{1, 2, 3, 4, 5\}$. Find a subbase for this topology. [BTL3]
5. Define connectedness and path connectedness. [BTL1]
6. Define a compact space and prove that a cofinite topological space is compact. [BTL1]
7. Prove that in a T_2 space the limits of sequences are unique [BTL1]
8. Prove that there exist no countable connected T_3 space [BTL3]

Part B

*Answer **any two** questions from each module. Weightage **2** for each question.*
(6x2 = 12 Weightage)

Unit-I

9. Let X be a non empty set and the collection $\{\mathbb{T}_i : i \in I\}$ be an indexed family of topologies on X. Let $\mathbb{T} = \bigcap_{i=1}^{n} \mathbb{T}_i$. Then prove that $\mathbb{T}$ is a topology and it is weaker than each $\mathbb{T}_i$. [BTL2]
10. Define accumulation point of a subset A of a topological space X and deduce that the set of all integers has no accumulation point. [BTL3]
11. a) Define boundary of a set A. Find the boundary of the compliment of A. [BTL4]
    b) Prove that a set A is clopen in a topological space if and only if it has an empty boundary.

Turn Over

Unit-II

12. Prove that compactness is an absolute property. [BTL2]

13. Give an example of a countable space which is not second countable. [BTL4]

14. Let $\mathbb{C}$ be a collection of connected subsets of a space X such that no two members of $\mathbb{C}$, are mutually separated. Then show that $\cup_{E\in\mathbb{C}} E$ is connected. [BTL3]

Unit-III

15. Suppose y is an accumulation point of a subset A of a T_1 space X. Then deduce that every neighbourhood of y contains infinitely many points of A. [BTL3]

16. Prove that every Tychonoff space is T_3. [BTL2]

17. (a) Prove that every regular Lindeloff space is normal. [BTL4]
    (b) Prove that every compact Hausdorff space is T_4.

Part C

*Answer **any two** questions. Weightage **5** for each question.* ***(2x5 = 10 Weightage)***

18. a) If a space is second countable, prove that every open cover of it has a countable subcover. [BTL3]
    b) For a topological space $(X, \mathbb{T})$, let $\mathbb{S}$ be a family of subsets of X. Then prove that $\mathbb{S}$ is a subbase for $\mathbb{T}$ if and only if $\mathbb{S}$ generates $\mathbb{T}$.

19. Prove that every continuous real valued function on a compact space is bounded and attains its extrema. [BTL4]

20. State and prove Lebesque Covering Lemma. [BTL4]

21. State and prove Uryshon Lemma. [BTL2]

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