QP Code: U25B026

Reg. No : .....

Name : .....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

II SEMESTER (FYUGP) DEGREE EXAMINATION, MARCH 2025
B.A/B.Sc./B.Com/BSW
MAT2MN103 : ANALYSIS AND SOME COUNTING PRINCIPLES
2024 Admission Onwards
(Credits: 4)

Time: 2 Hours **Maximum Marks: 70**

Section A

*Answer **all**. Each question carries **3** Marks* ***(Ceiling: 24 Marks)***

1. Write the first five terms of the recursively defined sequence $a_1 = 3$, $a_{k+1} = 2(a_k - 1)$ [BTL4]
2. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$ [BTL2]
3. Define a Monotone Sequence and give an example. [BTL1]
4. Evaluate i^{12}. [BTL3]
5. Express $z = \sqrt{3} - i$ in polar form. [BTL4]
6. Define a region in the complex plane. [BTL1]
7. Find the derivative of $f(z) = -5iz^2 + \frac{2+i}{z^2}$. [BTL5]
8. Show that the Cauchy- Riemann equations are satisfied for the function $f(z) = 3z^2 + 5z - 6i$ at every point. [BTL3]
9. Compute each of the following [BTL3]
   i) ${}^6\mathrm{p}_5$
   ii) ${}^7\mathrm{C}_7$
10. Define the frequency of occurence of an event. [BTL2]

Section B

*Answer **all**. Each question carries **6** Marks* ***(Ceiling: 36 Marks)***

11. Show that the sequence $\{C_n\} = \{(-1)^n \frac{1}{n!}\}$ converges and find its limit. [BTL4]
12. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{3n^2-4n+5}$ [BTL2]

Turn Over

13. i) Find the modulus of the complex number $z = i(2 - i) - 4(1 + \frac{1}{4}i)$ [BTL4]
ii) Let $z = x + iy$. Express $|z + 5\bar{z}|$ in terms of x and y

14. Write the complex numbers $(cos\frac{\pi}{9} + isin\frac{\pi}{9})^{12}$, $[2(cos\frac{\pi}{6} + isin\frac{\pi}{6})]^5$ in polar form and in $a + ib$ form. [BTL3]

15. Find the real and imaginary parts of the functions [BTL2]
i) $f(z) = z^2 - (2 + i)z$
ii) $g(z) = z + 2Re(z)$

16. Verify that the function $u(x, y) = x^3 - 3xy^2$ is harmonic in the appropriate domain D. [BTL4]

17. i) A bank password consists of two letters of the English alphabet followed by two bdigits. How many different passwords are there? [BTL3]
ii) A catered menu is to include a soup, a main course, a dessert, and a beverage. Suppose that a customer can select from four soups, five main courses, three desserts and two beverages. How many different menus can be selected?

18. i) State the Extended Pigeonhole Principle. [BTL5]
ii) Show that if seven colors are used to paint 50 bicycles, at least 8 bicycles will be the same color.

Section C

*Answer **any one**. Each question carries **10** Marks* ***(1x10=10 Marks)***

19. a) Simplify the ratio of factorial $\frac{(n+1)!}{(n-1)!}$ [BTL5]
b) Find the sequence of partial sums S_1, S_2, S_3, S_4 and S_5 of the series $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}}$
c) Apply integral test to the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

20. i) Sketch the graph of the equation $|z + 2 + 2i| = 2$ in the complex plane. [BTL3]
ii) Find the fourth roots of $z = 1 + i$

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