QP Code: P25C017

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

III SEMESTER (CBCSS -PG) DEGREE EXAMINATIONS, OCTOBER 2025
CBCSS - M. Sc. Mathematics
MTH3C12 : COMPLEX ANALYSIS
2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

Answer ***all*** *questions. Weightage* ***1*** *for each question.* ***(8x1 = 8 Weightage)***

1. Let $\gamma : [a, b] \to \mathbb{C}$ be of bounded variation. If P and Q are partitions of [a,b] and $P \subset Q$ then prove that $V(\gamma; P) \leq V(\gamma; Q)$.
2. Prove that $|exp(z)| = exp(Rez)$.
3. Prove that a branch of the logarithm function is analytic and its derivative is $\frac{1}{z}$.
4. Prove that z is symmetric to itself with respect to Γ if and only if $z \in \Gamma$.
5. Let f and g be analytic on a region G. Prove that $f \equiv g$ if and only if $\{z \in G : f(z) = g(z)\}$ has a limit point in G.
6. Let γ and σ be closed rectifiable curves having the same initial points. Prove that $n(\gamma + \sigma; a) = n(\gamma; a) + n(\sigma; a) \forall a \notin \{\gamma\} \cup \{\sigma\}$.
7. Find the number of zeros of $f(z) = z^4 - 5z + 1$ within the annulus $1 < |z| < 2$.
8. Determine the type of singularity of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at $z = -2$.

Part B

Answer ***any two*** *questions from each module. Weightage* ***2*** *for each question.*
(6x2 = 12 Weightage)

Unit-I

9. State and prove Fundamental theorem of calculus.
10. State and prove Symmetry Principle.
11. Let G and Ω be open subsets of $\mathbb{C}$. Suppose that $f : G \to \mathbb{C}$ and $g : \Omega \to \mathcal{C}$ are continuous functions such that $f(G) \subset \Omega$ and $g(f(z)) = z, \forall z \in G$. If g is differentiable and $g'(z) \neq 0$, prove that f is differentiable and $f'(z) = \frac{1}{g'(f(z))}$.

Turn Over

Unit-II

12. State and prove Morera's theorem.

13. Let $f : G \to \mathcal{C}$ be analytic and suppose $\overline{B(a,r)} \subset G$. Let $\gamma(t) = a + re^{it}, 0 \leq t \leq 2\pi$, prove that $f(z) = \frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w-z} dw, |z-a| < r$.

14. Let G be a region and let f be an analytic function on G with $a_1, a_2, \ldots, a_m$ are the points that satisfy the equation $f(z) = \alpha$. Let γ is a closed rectifiable curve in G which does not pass through any point $a_k$and $\gamma \approx 0$, prove that $\frac{1}{2\pi i} \int_\gamma \frac{f'(z)}{f(z)-\alpha} = \sum_{k=1}^{m} n(\gamma; a_k)$.

Unit-III

15. Let G be a region in $\mathbb{C}$ and f an analytic function on G. Suppose there is a constant M such that $\lim_{z\to a} sup|f(z)| \leq M, \forall a \in \partial_\infty G$. Prove that $|f(z)| \leq M, \forall z \in G$.

16. If G is a region with $a \in G$ and if f is analytic on $G - \{a\}$ with a pole at $z = a$. Prove that there is a positive integer m and an analytic function $g : G \to \mathbb{C}$ such that $f(z) = \frac{g(z)}{(z-a)^m}$.

17. State and prove Argument Principle.

Part C

*Answer **any two** questions. Weightage **5** for each question.* ***(2x5 = 10 Weightage)***

18. Let the series $\sum_{n=0}^{\infty} a_n(z-a)^n$ has radius of convergence $R > 0$, prove that $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$ is analytic in $B(a,R)$.

19. State and prove Higher derivative lemma.

20. State and prove Goursat's theorem.

21. Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$.

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