QP Code: P25C032

Reg. No : .....

Name : .....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

III SEMESTER (CBCSS-PG) DEGREE EXAMINATIONS, OCTOBER 2025

M. Sc Mathematics

MTH3C14 : PDE & INTEGRAL EQUATIONS

2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Section A

Answer **all**. Weightage **1** for each. **(8x1 = 8 Weightage)**

1. Show that the Cauchy problem $u_x + u_y = 1,\ u(x,x) = x$ has infinitely many solutions.

2. Find a generalized solution of homogeneous wave equation in one dimension.

3. Solve the heat problem $u_t - u_{xx} = 0, \quad 0 < x < \pi,\ t > 0$
$u(0,t) = u(\pi,t) = 0, t \geq 0$ and $u(x,0) = f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$

4. If $u(x,y)$ is a harmonic function in a domain D, then show that $u \in C^\infty(D)$.

5. Define a harmonic function.Give an example.

6. Show that the characteristic functions corresponding to distinct characteristic numbers of $y(x) = \lambda \int_a^b K(x,\xi)y(\xi)d\xi$ are orthogonal.

7. Show that $\frac{d^n}{dx^n} I_n(x) = (n-1)! f(x),$ where $I_n(x) = \int_a^x (x-\xi)^{n-1} f(\xi)d\xi.$

8. Briefly explain the iterative procedure for solving a Fredholm integral equation of second kind.

Section B

Answer **any two** questions from each module. Weightage **2** for each.

(6x2 = 12 Weightage)

Unit - I

9. Solve the following Cauchy problem
$u_{tt} - 9u_{xx} = e^x - e^{-x} \quad -\infty < x < \infty,\ t > 0,$
$u(x,0) = x \quad -\infty < x < \infty,$
$u_t(x,0) = sin\ x \quad -\infty < x < \infty.$

Turn Over

10. Fix $T > 0$. Show that for $0 \le t \le T$, the Cauchy problem

$$u_{tt} - c^2 u_{xx} = F(x,t) \qquad -\infty < x < \infty, t > 0,$$
$$u(x,0) = f(x), \;\; u_t(x,0) = g(x) \qquad -\infty < x < \infty$$

is well posed for $F, F_x \in C(\mathbb{R}^2),\ f \in C^2(\mathbb{R}),\ g \in C^1(R)$

11. Solve $u_x + u_y + u = 1$ subject to the initial condition $u = sin\, x$ on $y = x + x^2,\ x > 0$ by checking the transversality condition.

Unit - II

12. Let v be a function in C_H satisfying $v_t - k\,\Delta v < 0$ in Q_T .Then show that v has no local maximum in Q_T and v achieves its maximum in $\partial_P Q_T$.

13. State the problem of a vibrating string of finite length (without external forces) and find its generalized solution.

14. Let D be a bounded domain and let $u \in C^2(D) \cap C(\bar{D})$ be a harmonic function in D.Then show that the minimum of u in $\bar{D}$ is achieved on the boundary ∂D.

Unit - III

15. Using Green's function, transform the differential equation $\frac{d^2y}{dx^2} + y = x, y(0) = y(1) = 0$ to a Fredholm integral equation.

16. By iterative method, solve the integral equation $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$

17. Solve by separable kernel method, the integral equation
$y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi$.

Section C

Answer **any two** questions. Weightage **5** for each. **(2x5 = 10 Weightage)**

18. Reduce to canonical form the PDE
$u_{xx} - 2\, sinx\, u_{xy} - cos^2 x\, u_{yy} - cosx\, u_y = 0$ and find the general solution.

19. By the characteristics method, solve $-yu_x + xu_y = u$ subject to the initial condition $u(x,0) = \psi(x)$.

20. Solve the following problem by the method of separation of variables

$$u_{tt} - c^2 u_{xx} = 0 \qquad 0 < x < L, \;\; t > 0,$$
$$u_x(0,t) = u_x(L,t) = 0 \qquad t \ge 0,$$
$$u(x,0) = f(x), \;\; u_t(x,0) = g(x) \qquad 0 \le x \le L\,.$$

21. Solve the integral equation $y(x) = 1 + \lambda \int_0^1 x\xi\, y(\xi) d\xi$ by the iterative method.

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