QP Code: P25C026

Reg. No : ....

Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

III SEMESTER (CBCSS-PG) DEGREE EXAMINATIONS, OCTOBER 2025

M. Sc. Mathematics

MTH3C13 : FUNCTIONAL ANALYSIS

2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each question.* ***(8x1 = 8 Weightage)***

1. Define a normed space with an example.
2. Show that the unit ball D(E) = $\{x : ||x|| \leq 1\}$, is convex.
3. Check if $\langle f, g\rangle = \int_a^b f(t)\bar{g(t)}$ is an innerproduct in $C_2[a, b]$
4. Show that l^2 is a Hilbert space.
5. If $\{f_i\}$ is a system of vectors in a Hilbert space H such that $x \perp f_i$ for any i $\implies$ $x = 0$, then prove that $\{f_i\}$ is a complete system.
6. Define annihilator of a subspace L of a normed space X.
7. Define bounded linear operator. Show that $L(X, Y)$, the space of all bounded linear maps from X to Y is a normed space.
8. Define total set. Show that X* is a total set.

Part B

*Answer **any two** questions from each module. Weightage **2** for each question.*
(6x2 = 12 Weightage)

Unit-I

9. Show that dim $E/E_1 = n$ if and only if there exists $x_1, x_2, \ldots, x_n$ linearly independent vectors relative to E_1 such that for every $x \in E$, there exists unique set of numbers $a_1, a_2, \ldots, a_n$ and a unique vector $y \in E_1$ such that $x = \sum_{i=1}^{n} a_i x_i + y$.
10. State and prove integral form of Holder's inequality.
11. If X_0 is a closed subspace of X, show that X/X_0 is a normed space.

Turn Over

Unit-II

12. If $f, g \in E^{\#}/\{0\}$, and $ker f = ker\ g$, show that f and g are linearly dependent.

13. Show that every vector $x \in H$ has a unique decomposition in terms of elements of L and $L^{\perp}$, for every closed subspace L of H.

14. State and prove Bessel's inequality.

Unit-III

15. Show that, there exists $f \in X^*$ satisfying $f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in X$, such that $x_1 \neq x_2$.

16. If A and B are invertible operators, prove that AB is invertible.

17. Show that there exists a continuous linear operator on C[0,1].

Part C

*Answer **any two** questions. Weightage **5** for each question.* ***(2x5 = 10 Weightage)***

18. Prove that for every sequence of scalars (a_i) and (b_i) and for $1 < p < \infty$, $|\sum a_k b_k| \leq (\sum |a_k|^p)^{\frac{1}{p}} (\sum |b_k|^q)^{\frac{1}{q}}$, where $\frac{1}{p} + \frac{1}{q} = 1$

19. State and prove Riesz representation theorem.

20. Show that any two seperable infinite dimensional Hilbert spaces H_1 and H_2 are isometrically equivalent.

21. Show that $l_1^* = l_\infty$.

****