QP Code: P25DRIS034

Reg. No : .....
Name : .....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER (CBCSS - PG) DEGREE EXAMINATIONS, NOVEMBER 2025
(Regular/Improvement/Supplementary)
M. Sc. Mathematics
MTH1C05 : Number Theory
2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each.* ***(8x1 = 8 Weightage)***

1. Prove that $\phi(mn) = \phi(m)\phi(n)$, if $\gcd(m, n) = 1$.
2. Find all integers n such that $\phi(n) = \frac{n}{2}$.
3. Show that $\phi(n)$ is even for $n \geq 3$. Moreover if n has r distinct odd prime factors, then $2^r / \phi(n)$.
4. Give an example for a multiplicative function which is not completely multiplicative.
5. If $f(x) = O(1)$ then prove that $f(x) = o(x)$.
6. If $x \geq 1$, prove that $\lim_{x\to\infty}(\frac{M(x)}{x} - \frac{H(x)}{x \log x}) = 0$.
7. Find the quadratic residue modulo 11.
8. Working in 26-letter alphabet,using digraph enciphering transformation $C \equiv 159\ P + 580\ (mod\ 676)$, encipher the word "ON".

Part B

*Answer **any two** questions from each module. Weightage **2** for each.*
(6x2 = 12 Weightage)

Unit-I

9. Prove that $H(x) = \sum_{n\leq x} f(n)G(\frac{x}{n}) = \sum_{n\leq x} g(n)F(\frac{x}{n})$ where $H(x) = \sum_{n\leq x} h(n)$, $h = f * g$, $F(x) = \sum_{n\leq x} f(n)$ and $G(x) = \sum_{n\leq x} g(n)$
10. If both g and $f * g$ are multiplicative, then prove that f is also multiplicative.
11. Prove that if $n \geq 1$, $\sum_{d/n} \wedge(d) = \log n$.

Turn Over

Unit-II

12. Prove that $\lim_{n\to\infty} \frac{P_n}{n \log n} = 1 \implies \lim_{x\to\infty} \frac{\pi(x) \log \pi(x)}{x} = 1.$

13. For $x > 0$, prove that $0 \leq \frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x} \log 2}.$

14. Prove that for $x \geq 1$, $\sum_{n \leq x} \frac{a(n)}{n} = log\ x + O(1)$ where $\{a(n)\}$ is a non-negative sequence such that $\sum_{n \leq x} a(n)[\frac{x}{n}] = x\ log\ x + O(x)\ \ for\ \ all\ \ x \geq 1.$

Unit-III

15. For every odd prime p, prove that $(2/p) = (-1)^{\frac{p^2-1}{8}} = 1, \quad \text{if } p \equiv \pm 1 (\mod 8)$
$= -1, \quad \text{if } p \equiv \pm 3 (\mod 8).$

16. Find the inverse of $A = \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \in M_2(\frac{Z}{26Z})$.

17. Determine whether -104 is a quadratic residue of the prime 997.

Part C

*Answer **any two** questions. Weightage **5** for each. **(2x5 = 10 Weightage)***

18. For $x \geq 2$, prove that $\sum_{p \leq x} [\frac{x}{p}] \log p = x \log x + O(x)$ where the sum is extended over all primes less than or equal to x.

19. Prove that prime number theorem $\psi(x) \sim x \implies \lim_{x\to\infty} \frac{M(x)}{x} = 0.$

20. Show that the Diophantine equation $y^2 = x^3 + k$ has no solutions if k has the form, $k = (4n-1)^3 - 4m^2$ where m,n are integers such that no prime $p \equiv -1 (mod 4)$ divides m.

21. You intercept the cipher text message "PWULPZTQAWHF" , which you know was encripted using an affine map on digraph in 26 letter alphabet. Most frequently occuring digraph in the cipher text are "IX" and "TQ" in that order. It is known that the most common digraphs in English language are "TH" and "HE" in that order. Find the deciphering key and read the message.

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