QP Code: P25DRIS017

Reg. No : .....
Name : .....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER (CBCSS - PG) DEGREE EXAMINATIONS, NOVEMBER 2025
(Regular/Improvement/Supplementary)
M. Sc Mathematics
MTH1C02 : Linear Algebra
2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each.* ***(8x1 = 8 Weightage)***

1. Define a linear transformation. Let V be a vector space of all polynomial functions of degree $\leq n$ over a field F. Then prove that the map $D : V \to V$ defined by $D(f) = \frac{df}{dx}$ is a linear transformation.

2. Let T be a linear operator on $\mathcal{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$Find a rule for its inverse function?

3. If A is an $m \times n$ matrix over F and B, C are $n \times p$ matrices over F then prove that $A(dB + C) = dAB + AC$ for each scalar d in F.

4. In a finite dimensional vector space V, prove that every non empty linearly independent set of vectors is part of a basis.

5. Let $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ be the real 3×3 matrix . Find the characterestic values of A.

6. Define annihilator of a subset of a linear space. Also prove that the annihilator of a subset S of a vector space V is a subspace of its dual space.

7. Define orthogonal set and orthonormal set of vectors and write the inequality connecting norm and inner product.

8. What are independent subspaces? If $W_1, W_2, \cdots, W_k$ are independent subspaces of a finite dimensional vector space V, then prove that for every j, $2 \leq j \leq k$, $W_j \cap (W_1 + W_2 + \cdots + W_{j-1}) = \{0\}$

Turn Over

Part B

*Answer **any two** questions from each module. Weightage **2** for each.*

(6x2 = 12 Weightage)

Unit-I

9. Let V be a vector space over a subfield F of complex numbers. Suppose α, β, γ are linearly independent vectors in V. Prove that $(\alpha+\beta), (\beta+\gamma), (\alpha+\gamma)$ are linearly independent.

10. Prove that the subspace spanned by a non empty subset S of a vector space V is the set of all linear combinations of vectors in S.

11. Let V and W be vector spaces over the field F . Let T and U be linear transformations from V into W. Then prove that the function $(T+U)$ defined by $(T+U)(\alpha) = T\alpha + U\alpha$ is a linear transformation from V into W and if c is any element of F, the function (cT) defined by $(cT)(\alpha) = cT(\alpha)$ is a linear transformation from V into W.

Unit-II

12. Let V be a finite dimensional vector space over the field F. Then prove that each basis for V^* is the dual basis of some basis for V.

13. If W is a k dimensional subspace of an n dimensional vector space V, then prove that W is the intersection of $(n-k)$ hyperspaces in V.

14. Let W be an invariant subspace for T. Then prove that :
a) The characteristic polynomial for the restriction operator T_W divides the characteristic polynomial for T.
b) The minimal polynomial for T_W divides the minimal polynomial for T.

Unit-III

15. Write and prove the conditions for W to become the direct sum of subspaces $W_1, W_2, \cdots, W_k$ of an vector space V.

16. Consider the vectors $\beta_1 = (3,0,4), \beta_2 = (-1,0,7)$ and $\beta_3 = (2,9,11)$ in R^3 equipped with the standard innerproduct. Apply Gram-schimdt process to $\beta_1, \beta_2, \beta_3$ to find $\alpha_1, \alpha_2, \alpha_3$.

17. State and prove Bessel's Inequality.

Part C

*Answer **any two** questions. Weightage **5** for each.* ***(2x5 = 10 Weightage)***

18. Let V and W be finite-dimensional vector spaces over the field F such that $dimV = dimW$. If T is a linear transformation from V into W, then prove that the following are equivalent: (i) T is invertible. (ii)T is non-singular. (iii) T is onto, that is, the range of T is W.

19. State and prove Cayley Hamilton Theorem.

20. Let T be a linear operator on a finite dimensional vector space V. Let $c_1, c_2, \cdots, c_k$ be the distinct characteristic values of T and let W_i be the nullspace of $T - c_i I$. Then prove that the following are equivalent conditions:
    i. T is diagonalisable.
    ii. The characteristic polynomial for T is $f = (x - c_1)^d 1 \cdots (x - c_k)^d k$ and $dimW_i = d_i$ for $i = 1, 2, \cdots k$.
    iii. $dimW_1 + \cdots + dimW_k = dimV$.

21. State and prove Gram Schmidt Orthogonalisation Process.

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