QP Code: P25DRIS026

Reg. No : .....

Name : .....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER (CBCSS - PG) DEGREE EXAMINATIONS, NOVEMBER 2025

(Regular/Improvement/Supplementary)

M. Sc. Mathematics

MTH1C03 : Real Analysis I

2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each.* ***(8x1 = 8 Weightage)***

1. Show that set of all rational numbers is countable.
2. Define convex set. Prove that every closed ball is a convex set.
3. Check whether $d(x,y) = (x-y)^2$ is a metric on **R.**
4. Let f be a bounded real function on [a,b]. Show that Riemann integrability of f^3 on [a,b] implies the Riemann integrability of f on [a,b].
5. Show by a suitable example, that L'Hospitals' rule fails to be true for Complex valued functions.
6. Let f be defined on [a,b]. Show that, if f is differentiable at x in [a,b], then f is continuous at x.
7. Let γ be a curve in the complex plane, defined on [0, 2π] by $\gamma(t) = e^{2it}$. Prove that the length of γ is 4π.
8. Define equicontinuous family of functions. How is equicontinuity and uniform continuity related?

Part B

*Answer **any two** questions from each module. Weightage **2** for each.*

(6x2 = 12 Weightage)

Unit-I

9. If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of finite subcollection of $\{K_\alpha\}$ is nonempty, then show that $\cap K_\alpha$ is nonempty.
10. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.
11. If f is monotonic on (a,b). Show that the set of points of (a,b) where f is discontinuous atmost countable.

Turn Over

Unit-II

12. State and prove intermediate value property for real differentiable functions.

13. State and prove linearity and monotonicity properties of Riemann Stieltjes integrals.

14. Let f be defined on [a,b]. If f is differentiable and has local minimum at a point x in (a,b), then show that $f'(x) = 0$.

Unit-III

15. Suppose $\{f_n\}$is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then show that f is continuous on E.

16. Prove that sequence $\{f_n\}$ converges to f with respect to the metric on $\mathscr{C}(X)$, (set of all complex valued continuous, bounded functions on X) if and only if $f_n \to f$ uniformly on K.

17. Show by a suitable example that every convergent sequence need not contain a uniformly convergent subsequence.

Part C

*Answer **any two** questions. Weightage **5** for each.* ***(2x5 = 10 Weightage)***

18. (a) Show that Cantor set is perfect.
    (b) Show that every k-cell is compact.

19. (i) Suppose α increases on [a,b], $a \leq x_0 \leq b$, α is continuous at x_0, $f(x_0) = 1$ and $f(x) = 0$ if $x \neq x_0$. Prove that $f \in \mathscr{R}(\alpha)$ and $\int f d\alpha = 0$.
    (ii) Suppose $f \geq 0$, f is continuous on [a,b] and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0$ for all x in [a,b].

20. (a) State and prove Taylor's theorem.
    (b) Let f be monotonic on [a,b], and if α is continuous on [a,b], then prove that $f \in \mathscr{R}(\alpha)$.

21. (a)Suppose $\{f_n\}$ is a sequence of functions, differentiable on [a,b] and such that $\{f_n(x_0)\}$ converges for some point x_0 on [a,b]. If $\{f_n'\}$ converges uniformly that $\{f_n\}$converges uniformly on [a,b] to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x)$, a < x < b.
    (b) Show that there exists a real continuous function on the real line which is nowhere differentiable.

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