QP Code: P25DRIS032

Reg. No : ....

Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER (CBCSS - PG) DEGREE EXAMINATIONS, NOVEMBER 2025
(Regular/Improvement/Supplementary)
M. Sc. Mathematics
MTH1C04 : DISCRETE MATHEMATICS
2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each.*
(8x1 = 8 Weightage)

1. Explain a complemented bounded lattice.
2. Explain a Boolean function with an example.
3. Define join and meet of two elements of a partially ordered set $(X, \leq)$. Also, if x and y are comparable in X, then give $x \vee y$ and $x \wedge y$.
4. Define cut vertex and cut edge and give examples.
5. Define a planar graph and give an example.
6. Define graph isomorphism with an example.
7. Define an alphabet and a string with example.
8. Explain the language accepted by a dfa.

Part B

*Answer **any two** questions from each module. Weightage **2** for each.*
(6x2 = 12 Weightage)

Unit-I

9. i. Describe the diagonal ΔX of a set X.
ii. If R is a partial order on a set X, then prove that $R - \Delta X$ is a strict partial order on X.
10. Prepare the table of values of the Boolean function $f(x_1, x_2, x_3) = x_1 x_2 + x_2' x_3$
11. i. If X is a finite Boolean Algebra, then prove that $|X| = 2^n$ for some non negative integer n.

Turn Over

ii. Does there exist a Boolean Algebra with 9 elements. Justify your answer.

Unit-II

12. Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.

13. i) If $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$, $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and I_G is given by $I_G(e_1) = \{v_1, v_5\}$, $I_G(e_2) = \{v_2, v_3\}$, $I_G(e_3) = \{v_2, v_4\}$, $I_G(e_4) = \{v_2, v_5\}$, $I_G(e_5) = \{v_2, v_5\}$, $I_G(e_6) = \{v_3, v_3\}$. Then draw the graph $(V(G), E(G), I_G)$
ii) Define complement of a graph with an example.
iii) Define a self complementary graph with an example.

14. Prove that the Petersen graph P is non planar.

Unit-III

15. Find a grammar that generates $L = \{a^n b^{n+1} : n \geq 0\}$

16. Find a deterministic finite accepter that recognizes the set of all strings on $\sum = \{a, b\}$ starting with the prefix ***ab*.**

17. Show that all finite languages are regular.

Part C

Answer ***any two*** *questions. Weightage* ***5*** *for each question.* ***(2x5 = 10 Weightage)***

18. Let $(X, +, \cdot, ')$ be a Boolean algebra. Then, the corresponding lattice $(X, \leq)$ is complemented and distributive. Conversely, if $(X, \leq)$ is a bounded, complemented and distributive lattice, then there exists a Boolean algebra structure on X, $(X, +, \cdot, ')$ such that the partial order relation defined by this structure coincides with the given relation $\leq$.

19. i)Prove that the number of edges of a simple graph with ω components cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$.
ii)Prove that in a connected graph with atleast three vertices, any two longest paths have a vertex in common.

20. Let G be a planar graph. Then prove the following.
    i) All the plane embeddings of G have the same number of faces.

ii) If G is simple, the $\delta(G) \leq 5$.
iii)If G is connected and the girth $k \geq 3$, then $m \leq \frac{k(n-2)}{k-2}$.

21. Prove that, if u and v are strings, then the length of their concatenation is the sum of the individual lengths.

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