QP Code: P25DRIS008

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER (CBCSS - PG) DEGREE EXAMINATIONS, OCTOBER 2025
(Regular/Improvement/Supplementary)
M. Sc. Mathematics
MTH1C01 : Algebra- I
2024 Admission Onwards

Time: 3 Hours **Maximum Weightage: 30**

Part A

*Answer **all** questions. Weightage **1** for each.* ***(8x1 = 8 Weightage)***

1. Do the rotations, together with the identity map, form a subgroup of the group of plane isometries? Why or why not?
2. What is the order of the cyclic subgroup of $\mathbf{Z}_{24}$ generated by 18?
3. Show that if a finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G.
4. Let $\phi : \mathbf{Z_{12}} \rightarrow \mathbf{Z_3}$ be a homomorphism such that $\phi(1) = 2$. Find $Ker\phi$
5. Is group of order p^r for $r > 1$ simple ? Justify
6. Compute for the indicated evaluation homomorphism $\phi_i(2x^3 - x^2 + 3x + 2)$ where $F = E = \mathbf{C}$
7. Find all generators of the cyclic multiplicative group of units of the field $\mathbf{Z_{17}}$.
8. Find the multiplicative inverse of $2 - 2i + j + k$ in the quaternions of Hamilton

Part B

*Answer **any two** questions from each module. Weightage **2** for each.*
(6x2 = 12 Weightage)

Unit-I

9. Classify the group $(\mathbf{Z_2} \times \mathbf{Z_4})/ < (1,2) >$according to the fundamental theorem of finitely generated abelian groups.
10. How many distinguishable necklaces (with no clasp) can be made using seven different colored beads of the same size?
11. State and prove Fundamental Homomorphism Theorem.

Turn Over

Unit-II

12. Prove that every group of prime-power order is solvable.
13. State and prove second Sylow theorem.
14. State and prove second isomorphism theorem.

Unit-III

15. Let $\varphi : R \to R'$ be a ring homomorphism with kernel H. Prove that the additive cosets of H form a ring R/H whose binary operations are defined by $(a + H) + (b + H) = (a + b) + H$, and the product of the cosets is defined by $(a + H)(b + H) = (ab) + H$.
16. If $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_0$ is in $\mathbf{Z}[x]$ with $a_0 \neq 0$, and if $f(x)$ has a zero in Q, then prove that it has a zero m in Z, and m must divide a_0.
17. Give an example to show that a factor ring of a ring with divisors of 0 may be an integral domain.

Part C

*Answer **any two** questions. Weightage **5** for each.* ***(2x5 = 10 Weightage)***

18. Let G be a group,
    (a) Show that the commutator subgroup C is a normal subgroup of G.
    (b) Prove that if N is a normal subgroup of G, then G/N is abelian if and only if $C \leq N$.
19. If G is a finite group and p divides |G|, then show that the number of Sylow p-subgroups is congruent to 1 modulo p and divides |G|.
20. (a)State and prove Eisenstein criterion.
    (b) Show that $25x^5 - 9x^4 - 3x^2 - 12$i s irreducible over Q.
21. State and prove the Fundamental homomorphism theorem for rings

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