QP Code: U25DRIS034

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

I SEMESTER (FYUGP) DEGREE EXAMINATIONS, NOVEMBER 2025
(Regular/Improvement/Supplementary)
B. Sc. Mathematics
MAT1CJ101 : DIFFERENTIAL CALCULUS
2024 Admission Onwards
(Credits: 4)

Time: 2 Hours **Maximum Marks: 70**

Section A

*Answer **all**. Each question carries **3** Marks.* ***(Ceiling: 24 Marks)***

1. Describe the region defined by the inequality $(x-1)^2 + y^2 \leq 4$.
2. Find $\lim_{x \longrightarrow -2} \dfrac{3x+4}{x+5}$.
3. State Sandwich theorem.
4. If c is a constant then prove that $\dfrac{d}{dx}c = 0$.
5. At which points the function $f(x) = \frac{x+1}{x^2-4x+3}$ fails to be continuous.Are the discontinuities removable?
6. Find $\dfrac{d}{dx}(\sin^5 x)$.
7. Find all possible functions with derivative $y' = x^2$.
8. Find the value of c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$ for $f(x) = \sqrt{x-1} \;\; in \;\; [1, 3]$.
9. Find the points of inflection of the function $y = x^4 - 2x^2$.
10. Find $\lim_{x \longrightarrow -\infty} \dfrac{\pi\sqrt{3}}{x^2}$.

Turn Over

Section B

*Answer **all**. Each question carries **6** Marks.* ***(Ceiling: 36 Marks)***

11. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
1	$x-7$	$\sqrt{x}$	
2	$x+2$	$3x$	
3		$\sqrt{x-5}$	$\sqrt{x^2-5}$
4	$\dfrac{x}{x-1}$	$\dfrac{x}{x-1}$	
5		$1+\dfrac{1}{x}$	x
6	$\dfrac{1}{x}$		x

12. Suppose $\lim_{x \longrightarrow c} f(x) = 5$ and $\lim_{x \longrightarrow c} g(x) = -2$. Find

   1. $\lim_{x \longrightarrow c} f(x)g(x)$
   2. $\lim_{x \longrightarrow c} 2f(x)g(x)$
   3. $\lim_{x \longrightarrow c}(f(x) + 3g(x))$
   4. $\lim_{x \longrightarrow c} \dfrac{f(x)}{f(x) - g(x)}$

13. Differentiate the function $f(x) = \dfrac{8}{\sqrt{x-2}}$. Then find an equation of the tangent line at the point $(x, y) = (6, 4)$.

14. Find $\dfrac{dy}{dx}$ if $2y = x^2 + \sin y$.

15. Find the absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.

16. Show that hte function $f(x) = x^3 + \dfrac{4}{x^2} + 7$ has exactly one zero in $(-\infty, 0)$.

17. Suppose u and v are functions of x that are differentiable at $x = 0$ and that $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$, $v'(0) = 2$. Find the values of the following derivatives at $x = 0$;
$\dfrac{d}{dx}(uv)$, $\dfrac{d}{dx}(\dfrac{u}{v})$, $\dfrac{d}{dx}(\dfrac{v}{u})$ and $\dfrac{d}{dx}(7v - 2u)$.

18. Find the asymptotes of the graph of $f(x) = \dfrac{x^2 - 3}{2x - 4}$.

Section C

*Answer **any one**. Each question carries **10** Marks.* ***(1x10=10marks)***

19. (i). Graph the function $f(x) = \dfrac{x}{|x|}$, then find $\lim_{x \longrightarrow 0^-} f(x)$ and $\lim_{x \longrightarrow 0^+} f(x)$

(ii). Evaluate the following limits $\lim_{x \longrightarrow 0^+} \dfrac{1}{3x}$.

$\lim_{x \longrightarrow 0^-} \dfrac{5}{2x}$

20. (i) State and prove Mean value theorem.

(ii) Verify the hypothesis of the Mean value theorem for the function $f(x) = x^{\frac{2}{3}}$ on the interval [-1, 8].

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