QP Code: U25C065

Reg. No : ....
Name : ....

ST MARY'S COLLEGE (AUTONOMOUS), THRISSUR-20

III SEMESTER (FYUGP) DEGREE EXAMINATIONS, OCTOBER 2025
B. Sc. Mathematics
MAT3CJ202 : MATRIX ALGEBRA
2024 Admission Onwards
(Credits: 4)

Time: 2 Hours **Maximum Marks: 70**

Section A

*Answer **all**. Each question carries 3 Marks.* ***(Ceiling: 24 Marks)***

1. Given $u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$. Find $4u$, $(-3)v$ and $4u + (-3)v$

2. Explain the terms ; pivot position and pivot column.

3. For what values of h and k is the following system consistent

$$\begin{array}{rcrcl} 2x_1 & - & x_2 & = & h \\ -6x_1 & + & 3x_2 & = & k \end{array}$$

4. Determine if the given sets of vectors are linearly independent; $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$

5. Let $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and define $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ by $T(x) = Ax$. Find the images under T of $u = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $Av = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

6. Let $A = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$. Show that these matrices do not commute.

7. Solve the system

$$\begin{array}{rcrcr} 8x_1 & + & 6x_2 & = & 2 \\ 5x_1 & + & 4x_2 & = & -1 \end{array}$$

8. Define dimension of a subspace H. Give example.

Turn Over

9. Find the eigen values of $A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$

10. The characteristic polynomial of a 6×6 matrix is $\lambda^6 - 4\lambda^5 - 12\lambda^4$. Find the eigen values and their multiplicities.

Section B

*Answer **all**. Each question carries **6** Marks.* ***(Ceiling: 36 Marks)***

11. Let $a_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $a_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, $a_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$, $b = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$. Determine if b is a linear combination of a_1, a_2 and a_3.

12. i) For v_1, v_2, v_3 in $\mathbb{R}^m$, write the linear combination $3v_1 - 5v_2 + 7v_3$ as a matrix times a vector
ii) Let $A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$, $u = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$. Verify that $A(u+v) = Au + Av$

13. Describe the solutions of the following system in parametric vector form
$$\begin{array}{rcrcrcr} x_1 & + & 3x_2 & + & x_3 & = & 1 \\ -4x_1 & - & 9x_2 & + & 2x_3 & = & -1 \\ & - & 3x_2 & - & 6x_3 & = & -3 \end{array}$$

14. If a set contains more vectors than there are entries in each vector, then prove that the set is linearly independent

15. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exists.

16. Find a basis for the column space of the matrix $B = \begin{bmatrix} 1 & 0 & -3 & 5 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

17. Is $\lambda = 3$ an eigen value of $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

18. Prove that, if $n \times n$ matrices A and B are similar, then they have the same characteristic polynomial and hence the same eigen values with the same multiplicities.

Section C

*Answer **any one**. Each question carries **10** Marks.* ***(1x10=10marks)***

19. i) Suppose $T : \mathbb{R}^5 \longrightarrow \mathbb{R}^2$ and $T(x) = Ax$ for some matrix A and for each x in $\mathbb{R}^5$. How many rows and columns does A have

    ii) Given a scalar r, define $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ by $T(x) = rx$. Let $r = 3$, and show that T is a linear transformation

    iii) Given a linear transformation $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$. Prove that T is one-to-one if and only if the equation $T(x) = 0$ has only the trivial solution .

20. Diagonalize the following matrix, if possible $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

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