

**FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2009**

Mathematics—Complementary Course

MM IC 01—MATHEMATICS

(C.S.S.)

Time : Three Hours

Maximum Weightage : 30

**I. Objective Type Questions - Answer all 12 questions (12 x 1/4 = 3 weightage)**

1.  $P(x)$  and  $Q(x)$  are polynomials and  $Q(c) \neq 0$  then  $\lim_{x \rightarrow c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}$
2. 3.  $\lim_{x \rightarrow 2} \frac{f(x)-5}{x-2} = \text{then}$   $f'(2) = \dots$
3. The function  $f(x) = \frac{\cos x}{x}$  is not continuous at  $x = \dots$
4. The slope of the curve  $y = \frac{1}{x}$  at  $x=1$  equals ...
5. If  $\lim_{x \rightarrow 1} f(x) = \frac{1}{2}$ , then  $\lim_{x \rightarrow 1} \frac{f(x) \cos x}{x-1} = \dots$
6. The absolute maximum value of  $f(x) = -x - 4$ ,  $-4 \leq x$  is at  $x = \dots$
7. If  $f'(x) > 0$  for every  $x$  in  $(a, b)$ , then  $f$  is  $\dots$  in  $(a, b)$ .
8. The horizontal asymptote of the curve  $y = \frac{1}{x}$  is ...
9. If  $f$  is continuous and  $\int_1^2 f(x) dx = -4$  and  $\int_2^3 f(x) dx = 6$  then  $\int_1^3 f(x) dx = \dots$
10. If  $f$  is integrable on  $[a, b]$ , then the average value of  $f$  on  $[a, b]$  is  $\text{av}(f) = \frac{1}{b-a} \int_a^b f(x) dx$
12. If  $f$  is smooth on  $[a, b]$ , the length of the curve  $y = f(x)$  from  $a$  to  $b$  is  $L = \int_a^b \sqrt{1 + f'(x)^2} dx$

**II. Short answer type questions - Answer all 9 questions (9 x 1 = 9 weightage)**

13. If  $f(x) = 2x - 4$  and  $x_0 = 5$ ,  $E = 0.2$  and  $L = 6$ , find  $\delta > 0$ , such that  $0 < |x - x_0| < \delta$  implies  $|f(x) - f(x_0)| < E$ .

**Turn over**

14. For what values of  $a$  is  $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax - 1, & x \geq 3 \end{cases}$  continuous at  $x = 3$ ?
15. Find the value of  $c$  that satisfies the mean value theorem for the function  $f(x) = x^2 + 2x - 1$  on  $[0, 1]$ .
16. If  $x$  moves from left to right through the point  $c = 2$ , is the graph of  $f(x) = x^3 - 3x + 2$  rising or falling? Justify your answer.
17. Use Sandwich theorem to find the asymptotes of  $y = 2 + \frac{1}{x}$ .
18. Find the inflection point of the curve  $f(x) = x^3 - 3x + 3$ .
19. Consider the function  $f(x) = x^2 - 1$  on  $[0, 2]$ . Partition the interval into four subintervals of equal length. Find the Riemann sum  $\sum_{k=1}^4 f(c_k) \Delta c_k$  where  $c_k$  is the left end point.
20. State the mean value theorem for definite integrals.
21. Find the area between the curves  $y = \sec x$  and  $y = \sin x$  from 0 to  $\frac{\pi}{4}$ .

**III. Short essay or paragraph questions – Answer any 5 questions from 7 (5 x 2 = 10 weightage)**

22. Draw the graph of the function

$$f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ \frac{1}{x}, & x > 2 \end{cases}$$

Find the limits or explain why they do not exist?

a)  $\lim_{x \rightarrow 2^+} f(x)$

b)  $\lim_{x \rightarrow 2^-} f(x)$

c) Does  $\lim_{x \rightarrow 2} f(x)$  exist? If so what is it? If not, why?

23. If a function is differentiable at  $x = c$ , prove that it is continuous at  $x = c$ . Is the converse true? Justify your answer.
24. If  $b, c, d$  are constants, for what value of  $b$  will the curve  $y = x^3 + bx^2 + cx + d$  has a point of inflection at  $x = 1$ .
25. Find the average value of  $f(x) = 3x^2 - 3$  on  $[0,1]$ . At what points in the interval does this function assume its average value?
26. Find the area of the region between the curve  $y = 4 - x^2$ ,  $0$  and the  $x$ -axis.
27. The region between the curve  $y = \quad$ ,  $0 \leq x \leq 2$ , and the  $x$ -axis is revolved about the  $x$ -axis. Find the volume of the solid generated.
28. Applying L'Hospital's rule find  $\lim_{x \rightarrow \infty}$

**IV. Essay questions – Answer 2 questions from 3 (2 x 4 = 8 weightage)**

29. Sketch the graph of  $y = \sin(1/x)$  and show that  $y = \sin(1/x)$  has no limit as  $x$  approaches zero from either side.

Or

If  $f$  and  $g$  are continuous functions, then prove that

- a)  $f + g$  is continuous      b)  $fg$  is continuous

30. Plot the graph and find the derivative at each critical point and determine the local extreme values.

$$y = \begin{cases} -x^2 - 2x + 4, & x \leq 1 \\ -x^2 + 6x - 4, & x > 1 \end{cases}$$

31. Use definite integral to estimate the sum of the square roots of the first  $n$  positive integers,  $1 + \sqrt{2} + \dots + \sqrt{n}$ .

Or

Find the length of the curve  $y = x^{3/2}$  from  $x = 0$  to  $x = 4$ .