**D 74380** (Pages : 4) Name..................................

**Reg. No...............................**

**FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014**

(CUCBCSS-U.G.)

Complemental Course—Mathematics

MAT IC 01—MATHEMATICS

Three Hours Maximum : 80 Marks

**Section A**

*Answer all* **twelve** *questions.*

1. Evaluate Lt $\frac{x^2+3x-10}{x+5}$

2. At what points are the function Y = $\frac{\cos x}{x}$ continuous ?

3. Find the slope of $f(x) = x^2 + 1$ at (3, 7).

4. Find the derivative of y $x^2$ using the definition of derivative.

5. Find the second derivative of Y = $\frac{1}{3x2} - 5$

6.- How fast does the area of a circle change with respect to the diameter when the diameter is 8 m ?

7. Find the critical points of $f(x) = \frac{2x3 - 3x^2}{6}$.

8. Graph the parabola $y = x^2$.

9. Find $\operatorname{Lt}_{x\to\infty} \frac{2x+3}{5x+7}$.

10. Evaluate the sum of the first 20 cubes.

11. State the mean value theorem for definite integrals.

12. Find the intersection points of $f(x) = 2 - x^2$ and $g(x) = -x$.

(12 x 1 = 12 marks)

**Turn over**

## Section B

*Answer all* **nine** *questions.*

13. If $\overline{\quad 2x2} < f(x) < \overline{\quad -x^2}, -1 < x < 1$, find $\underset{x \to 0}{\text{Lt}} f(x)$

14. Prove that $\underset{x \text{-}3 1}{\text{Lt}} f(x) = 1$ if $f(x) = \begin{cases} x^2 & 1 \\ 2, & x = 1 \end{cases}$.

15. Suppose Lt $f(x) = 5$ and Lt $g(x) = -2$. Find :

(i) $\underset{\to c}{\text{Lt}} [f(x) + \;\; (x)]$ ; and (ii) $\frac{(x)}{f(x) - g(x)}$

16. Find the derivative of $Y = \dfrac{1}{(x^2 - -1)\,(x^2 + x + 1)}$

17. Find the equation of the tangent to the curve $y = x^3 - 4x + 1$ at (2, 1).

18. Find $\underset{x \to 0}{\text{Lt}} \dfrac{8x^2}{\cos x - 1}$

19. Find the linearization of $1(x) = x^3 - x$ at $x = 1$.

20. Graph the function $Y = \dfrac{1}{2x + 4}$

21. Find the area of the region enclosed by $y = x^2 - 2$ and $y = 2$.

22. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through (0, 2).

23. Find the derivatives of all orders of $\dfrac{5}{120}$.

24. State both parts of the fundamental theorem of calculus.

(9 x 2 = 18 marls)

## Section C

*Answer any* **six** *questions.*

25. Show that $= \sin\left(\frac{1}{x}\right)$ .has no limit point as $x$ approaches zero from either side. Also ketch the graph of this function.

26. Evaluate $\underset{x \to}{\text{Lt}} \dfrac{x-1}{\sqrt{x+3}-2}$

27. The curve $y = ax^{2} + bx + c$ passes through (1, 2) and is tangent to y = x at the origin. Find a, *b, c.*

28. State and prove the product rule for derivatives. Use it to find the derivative of $y = (x^2 + 1)(x^3 + 3)$.

29. Find the intervals on which $f(x) = \dfrac{x^2}{x-2}$, $x \neq 2$ is increasing and decreasing. Identify local extrema if they exist.

30. Define average value of an integrable function over a closed interval. Find the average value of $f(x)$ $3x^2 - 1$ on [0,1]. where in the given interval does $f(x)$ assume its average value.

31. Show that $\text{Lt} \left(1 + \frac{}{}\right) = e$.

32. An object is dropped from the top of a 100 m high tower. Its height above ground after '$t$' seconds is $(100 - 4.9\,t^2)$ $m$. How fast is it falling 2 seconds after it is dropped ?

33. Find the derivative $\dfrac{d}{dx}\int_0 \cos t\, dt$ by (i) evaluating the integral and differentiating the result ; and (ii) by differentiating the integral directly.

(6 x 5 = 30 marks)

## Section D

*Answer any* two *questions.*

34. (i) Find the area of the region enclosed by the curves $x + 4y^2 = 4$ and $x + y^4 = 1$ for $x > 0$.

    (ii) Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $y = 0$, $x = 2$ about the x-axis.

35. (i) Graph the function $y = x^4 - 4x^3 + 10$ by finding the first and second derivative.

    (ii) Evaluate $\underset{x \to 0}{\text{Lt}} \dfrac{x^2}{\ln(\sec x)}$.

    (iii) Evaluate $\sum_{k=1}^{4} \cos k\pi$

Turn over

36. (i) Let $f(x) = \begin{cases} 3-x, x<2 \\ \frac{x}{2}+1, x>2 \end{cases}$. Find :

(a) $\underset{x \to 2+}{\mathrm{Lt}} f(x)$ and $\underset{x \to 2-}{\mathrm{Lt}} 1(x)$

(b) Does $\underset{x \quad 2}{\mathrm{Lt}} f(x)$ exist ? why or why not ?

(c) $\underset{x \to 4-}{\mathrm{Lt}} 1(x)$ and $\underset{x\text{-}34+}{\mathrm{Lt}} 1(x)$

(d) Does $\underset{x \quad 4}{\mathrm{Lt}} \quad (x)$ exist ? Why or why not ?

(ii) Show that the line y = $mx + b$ *is* its own tangent at any point $(x_0, mx_0 + b)$.

(2 x 10 = 20 marks)