

C 15751

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Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2011

(CCSS)

Complementary Course

*poly* MM 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

I. Objective Type Questions. Answer *all* twelve questions :

1 Define hyperbolic cosine function in terms of exponential function.

2 If  $f$  is continuous on  $[a, c)$ , then,  $\lim_{x \rightarrow c^-} f(x) dx =$  \_\_\_\_\_

3 The derivative of  $\sec h 2x$  with respect to  $x$  is = \_\_\_\_\_

4 The value of  $\sinh^{-1} 1$  using logarithm is \_\_\_\_\_

5 When does a sequence of real numbers  $\{a_n\}$  converge to the number  $L$ .

6 The  $n^{\text{th}}$  term of the sequence 0, 1, 2, 2, 3, 3, 4, 4, is \_\_\_\_\_

7 Find  $\sum_{n=0}^{\infty} \binom{1}{3} 2^n$ .

8 The least upper bound of the sequence  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots$  is \_\_\_\_\_

9 Define absolute convergence of the series  $\sum a_n$ .

10 Define a power series about  $x = a$ .

11 The cylindrical co-ordinates of  $(0, 1, 0)$  (in Cartesian coordinate) is \_\_\_\_\_

12 State the chain rule for functions of two independent variables.

(12 x  $\frac{1}{4}$  = 3 weightage)

II. Short Answer questions. Answer *all* nine questions :

13 Evaluate the integral  $\int \sinh 2x dx$ .

14 Use partial fractions to find the sum of the series  $\sum_{n=1}^4 \frac{1}{(4n-3)(4n+1)}$  converges.

Turn over

15 Show that  $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$ .

16 Determine the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{12n-1}$ .

17 Prove or disprove that  $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$  converges.

18 State Rearrangement theorem for absolutely convergent series.

19 Define boundary point. Give an example.

20 For what values of  $x$  does the following power series converge ?

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{2} + \sum_{n=0}^{\infty} \frac{x^{3n}}{3}$$

21 Find the Taylor series expansion of  $f(x) = e^x$  at  $x = 0$ .

(9 x 1 = 9 weightage)

III. Short essay questions. Answer any *five* questions :

22 Show that  $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ ,  $- \infty < x < \infty$ .

23 Determine the convergence or divergence of the series  $\sum_{n=1}^{\infty} \left(\frac{1}{n^2}\right)$ .

24 State term by term differentiation theorem. Express  $f(x) = \frac{1}{1-x}$ ,  $|x| < 1$  as a power series.

Use the theorem to find  $f'(x)$  and  $f''(x)$ .

25 Find the length of the asteroid  $x = \cos^3 t$ ,  $y = \sin^3 t$ ,  $0 < t < 2\pi$ .

26 Graph the function  $r^2 = \sin 2\theta$ .

27 Find the  $f_x$ ,  $f_y$  and  $f_z$  for the function  $f(x, y, z) = x^2 - y^2 + z^2$ .

28 Find the linearization  $L(x, y, z)$  of the function  $f(x, y, z) = e^x + \cos(y + z)$  at  $(0, 0, 0)$ .

(5 x 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29 (a) Test for absolute convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin n$ .

(b) Identify the function :

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots; -1 < x < 1.$$

30 (a) Find the area of the region in the plane enclosed by the cardioid  $r = 2(1 + \cos \theta)$ .

(b) Find the spherical co-ordinate equation for the sphere  $x^2 + y^2 + (z-1)^2 = 1$ .

31 (a) If  $f(u, v, w)$  is differentiable and  $u = x - y$ ,  $v = y - z$  and  $w = z - x$  then show that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$$

(b) Show that  $f(x, y, z) = x^2 + y^2 - 2z^2$  satisfies the Laplace equation.

(2 x 4 = 8 weightage)